

## Worksheet for Sections 10.1 and 10.2

1. Consider the point  $P = (x, y)$  with rectangular coordinates  $(1, -\sqrt{3})$ . Let  $(r, \theta)$  be polar coordinates of  $P$ , with  $r > 0$  and  $0 \leq \theta < 2\pi$ .

- (a) Draw  $P$  in the usual Cartesian plane. Then find the values of  $r$  and  $\theta$ , giving reasons.  
(b) For the values of  $r$  and  $\theta$  found in part (a), draw on the same large graph (different from the graph in part (a)) the points with the following polar coordinates and label each:

$$(r, -\theta), \quad (r, \pi - \theta), \quad (r, \pi + \theta), \quad (-r, \theta), \quad (-r, \pi - \theta), \quad (-r, \pi/2 - \theta)$$

2. (a) Plot the graphs of  $r = \sin(7\theta)$  and  $r = \sin(4\theta)$ , for  $0 \leq \theta \leq 2\pi$ . Which rose is traced out twice, and which rose is traced out once as  $\theta$  increases from 0 to  $2\pi$ .

- (b) Let  $n$  be a positive integer, and consider the graph of  $r = \sin(n\theta)$ . Determine the number of leaves when  $n$  is odd, and the number of leaves when  $n$  is even. (The reason there is a difference between even  $n$  and odd  $n$  is interesting. Can you provide the reason?)

- (c) Find a polar equation for a 12-leaved rose.

3. Suppose that you are asked to sketch the graph of  $r = 2\cos(5\theta)$ , *without* the help of a calculator. Discuss how you would proceed. Indicate what values of  $\theta$  you would use to assist you in plotting significant points on the graph, and indicate what happens on the graph between successive significant points. Then sketch the graph of  $r = 2\cos(5\theta)$ .

4. Consider the lemniscate  $r^2 = 4\sin(2\theta)$ , for  $0 \leq \theta \leq 2\pi$ .

- (a) Sketch the lemniscate.

- (b) For what values of  $\theta$  in  $[0, 2\pi]$  is there no real value of  $r$ ? Indicate these values of  $\theta$  on the graph in part (a).

- (c) Find  $\int_0^{2\pi} \frac{1}{2}(4\sin(2\theta)) d\theta$ . Evidently the value of the integral is not the area  $A$  of the region enclosed by the lemniscate. Why? What is wrong with the given integral?

- (d) Find the area  $A$  enclosed by the lemniscate. (Hint: Choose limits of integration judiciously.)

5. Write down a formula in polar coordinates for a function whose graph has the given symmetry, and draw the graph of the function.

- (a) symmetry with respect to only the origin  
(b) symmetry with respect to only the  $x$  axis  
(c) symmetry with respect to only the  $y$  axis