Worksheet for Sections 6.1

NOTE: Problems 1, 2, 3 and 6 concern volumes (6.1). Problems 4 and 5 concern review.

1. Consider the solid region whose base R is bounded by the negative x axis, the positive y axis, and the curve $y = 4 - x^2$ for $-2 \le x \le 0$.

   (a) Let D_1 be the solid with base R, and assume that the cross sections of D_1 perpendicular to the x axis are squares. Draw a picture of the base, and then draw a representative cross section (perpendicular to the x axis) at some arbitrary x in the interval $(-2, 0)$. Finally, find the cross-sectional area A_1 of the cross section.

   (b) Now let D_2 be the solid with base R, and assume that the cross sections of D_2 perpendicular to the x axis are semi-circles. Draw a second picture of the base, and then draw a representative cross section (perpendicular to the x axis) at some arbitrary x in the interval $(-2, 0)$. Finally, find the cross-sectional area A_2 of the cross section.

   (c) Suppose that the solid D_3 had the same base R, but had equilateral triangles for cross sections perpendicular to the x axis. Without necessarily calculating the volumes, order the volumes V_1, V_2, V_3 of D_1, D_2, D_3 from least to greatest.

2. Let f and g be continuous and $0 \le g(x) \le f(x)$ on $[a, b]$. The washer method yields the volume formula $V = \int_a^b \pi[(f(x))^2 - (g(x))^2]\, dx$ when the region between the graphs of f and g is revolved about the x axis. Suppose $g(x) < 0 < f(x)$ and $|g(x)| \le |f(x)|$ on $[a, b]$. What should be the resulting formula for V when the bounded region between the graphs of f and g on $[a, b]$ is revolved about the x axis? Justify your answer.

3. (This problem is related to problem 44 in Section 6.1.) Consider a sphere of radius r that is centered at the origin. Take a vertical slice through the sphere at a distance h to the right of the center.

   (a) Draw a picture of the situation, and then show that the volume V of the smaller piece cut off satisfies the formula
   $$V = \frac{\pi(r-h)^2}{3}(2r + h)$$

   (b) Suppose that $r = 2$. Approximate the value of h such that V = half the volume of the right hemisphere, that is, such that $V = 8\pi/3$. (Hint: You likely will need to use the Newton-Raphson method to solve the equation.)

4. A cylindrical tank with a volume of 40π cubic meters is to be built. Materials for the sides cost 10 dollars per square meter, and for the top and bottom the materials cost 25 dollars per square meter. Find the dimensions that will minimize the cost.

5. Let $g(x) = \int_x^{x+\pi} \sin^{2/3} t\, dt$. Show that g is a constant function.

6. (if time) Do Problem 10 in the Review for Chapter 6, on p. 426 of the text.)