Worksheet for Section 6.4

- 1. In calculating the work W necessary to stretch a spring from its natural length of 8 centimeters to a length of 12 centimeters, we need some more information about the spring. There are two distinct kinds of information, each of which separately would provide the necessary assistance so we could find W. Describe what the two additional kinds of information are.
- 2. We wish to empty a hemispherical tank full of water out the top. Which should entail the larger amount of work: when the tank is like a cup full of water, or when the tank is upside down? Explain your answer, and include a picture of the situation.
- 3. In Example 4 (p. 393), a hemispherical tank with radius 10 feet is filled with water (Figure 6.43). In determining the work W required to pump all the water to 6 feet above the top of the tank, we assumed that the origin was at the *top* of the tank and derived the formula

$$W = \int_{-10}^{0} 62.5(6-x)\pi(100-x^2) \, dx$$

Now let us assume that the origin is at the *bottom* of the tank. Find a formula for the radius r(x) of the cross section x units above the origin, and use this formula to write down the integral for the work W. (If you have time, you should check that the two formulas for W yield the same number of foot-pounds.)

- 4. A painter weighing 120 pounds is hoisted by means of a rope windlass from the ground to a window on the tenth story of a hotel, 80 feet from ground level.
 - (a) If the weight of the entire rope is considered to be negligible, then find the work W required to lift the person the 80 feet from ground level.
 - (b) Assume that instead of a rope, a substantial chain weighing 2 pounds per foot is used in the windlass. Find the work W_1 required to lift the person the 80 feet from ground level. (Hint: As the person is lifted, the length of the chain that must be lifted decreases.)
- 5. (a) At a party, the punch bowl is hemispherical, with 1 foot radius and open at the top. The fruit punch weighs 66 pounds per cubic foot. At the beginning of the party the bowl is full to the brim. The rules for serving punch require each person to fill the ladle, raise it exactly 2 feet above the top of the bowl (without spilling!), and then pour the punch into a cup at that level. After 30 minutes, 40 people have filled 60 cups, and the punch remaining in the bowl is 4 inches deep. Draw a picture of the situation, and then set up the integral for the work W involved until the punch is only 4 inches deep. Include all pertinent information in the integral, but do not evaluate the integral.
 - (b) Suppose that each person is to raise the ladle exactly 2 feet (*not* 2 feet above the top of the bowl). Set up the integral for the work W involved until the punch is 4 inches deep. (Does this activity make any sense for the partygoers? Explain why or why not.)