Worksheet for Section 6.5

1. Let $f(x) = 3\sin x$ for $0 \le x \le \pi$, and consider the region R bounded by the x axis and the graph of f.

   (a) Explain in a complete sentence why you don't need to evaluate any integrals in order to determine the x coordinate $\overline{x}$ of the center of gravity.

   (b) Find the center of gravity $(\overline{x}, \overline{y})$ of R.

2. Let $r > 0$, and let R be the semicircular region bounded below by the x axis and above by the circle $x^2 + y^2 = r^2$, that is, $x^2 + y^2 = r^2$ with $y \ge 0$.

   (a) Find the center of gravity $(\overline{x}, \overline{y})$ of R.

   (b) Find the radius r for which $(\overline{x}, \overline{y}) = (0, \pi)$.

   (c) Suppose S is the quarter circular region in the first quadrant, bounded by the x axis, the y axis, and the quarter circle $x^2 + y^2 = r^2$. Without evaluating any integrals or making further computations, use symmetry considerations and computations in (a) to determine $(\overline{x}, \overline{y})$.

3. Let $a > 0$ and $h > 0$. Consider the triangular region R whose vertices are $(0, 0)$, $(a, 0)$, and $(0, h)$.

   (a) Find a formula for the line that represents the hypotenuse of the triangle.

   (b) Show that the center of gravity of R is $(a/3, h/3)$.
   (The center of gravity of R turns out to be the centroid of the triangle. However, you need to find the center of gravity of R in this problem by the method of Section 6.5.)

4. Let f and g be continuous on $[a, b]$, with $a > 0$ and $0 \le g(x) \le f(x)$ for $a \le x \le b$. Also let R be the region between the graphs of f and g on $[a, b]$, and A the area of R.

   (a) Write down a formula for the volume V of the solid obtained by revolving the region R about the y axis. (Note: If you are in doubt of the formula, please consult Section 6.1.)

   (b) Write down a formula for the moment M_y of R about the y axis.

   (c) Use the formulas in (a) and (b) to derive the formula $V = 2\pi\overline{x}A$. (This is the result of Pappus and Guldin in Theorem 6.6, where b is substituted for $\overline{x}$.)

   (d) Suppose someone says that $\overline{x} = 0$. What hypothesis in the problem helps to prevent $\overline{x} = 0$?

5. Find the center of gravity $(\overline{x}, \overline{y})$ of the region R in Exercise 32(d) in Section 6.5, which region can be split into two rectangles. Then use the formulas at the beginning of Exercise 32 to help find $(\overline{x}, \overline{y})$.