Worksheet for Section 7.1

Warmup:

1. Simplify the following: (a) $(\ln 2 + \ln 3)/\ln 36$ (b) $e^{-2\ln x}$ (c) $\ln(e^{(-x^2)})$

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2. Let $f(x) = mx + b$, where $m \neq 0$. Show that f has an inverse, and find a formula for f^{-1}. Finally, explain why f does *not* have an inverse if $m = 0$.

3. (a) Suppose that the formula $y = f(x)$ defines a function f that has an inverse on $[a, b]$. Describe the 2 steps you would use in order to find a formula for the inverse f^{-1}.
   (b) Let $f(x) = 3/x$. Draw the graph of f , then find a formula for f^{-1}, and conclude that $f^{-1} = f$. Do the same for $g(x) = -3/x$.
   (c) What property of the graph of a function h implies that $h^{-1} = h$?

4. (a) In a complete sentence, tell why an increasing function f defined on an interval I must have an inverse. Must the inverse be an increasing function? Explain your answer.
   (b) Draw the graph of a function f defined on the interval $[0, 1]$ that has an inverse but is *neither* increasing on $[0, 1]$ *nor* decreasing on $[0, 1]$.
   (c) Suppose that g is defined and differentiable on $[0, 2]$, with $g(1) = 3$. Assume that g has an inverse and $g'(1) = 0$. Can $(g^{-1})'(3)$ exist? Explain why or why not.

5. The function $f(x) = x^2$ has no inverse. However, if we restrict the domain to, say, $[0, \infty)$, then the new function f_1 *does* have an inverse. In the following, find the largest interval containing 0 for which the restricted function f_1 has an inverse:

   (a) $f(x) = \sin x$ (b) $f(x) = \tan x$ (c) $f(x) = x^3 - x$ (d) $f(x) = x + \sin x$

6. Suppose a ball is thrown vertically (either upward or downward) from a balcony h_0 meters above the ground, with an initial velocity v_0 and constant acceleration due to gravity (-9.8 meters per second squared). Assume further that the ball hits the ground after c seconds.

   (a) Write down a formula for the height h as a function of time t, for $0 \leq t \leq c$. (*Hint:* See p. 323.)
   (b) Under what physical conditions does h have an inverse h^{-1}?
   (c) In the event that h does have an inverse, what does h^{-1} tell us in physical terms?