Worksheet for Section 7.6

1. (a) Show that 1^∞ is an indeterminate form by finding 2 values of c for which $\lim_{x\to\infty}(1+\frac{c}{x})^x$ produces two different numbers.

   (b) Show that 0^∞ is a determinate form by looking at $\lim_{x\to\infty} f(x)^{g(x)}$, where $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to\infty} g(x) = \infty$, and telling the behavior of the values of $f(x)^{g(x)}$ as x approaches ∞.

2. Exercise 51 in the text asks us to evaluate one of the limits that appeared in l'Hôpital's original text on calculus: $\lim_{x\to a} \dfrac{a^2 - ax}{a - \sqrt{ax}}$, with $a > 0$.

   (a) Use l'Hôpital's Rule to evaluate the limit.

   (b) Rewrite the numerator as the difference of two squares, and simplify the resulting rational expression. Then evaluate the limit *without* using l'Hôpital's Rule.

3. Consider $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$.

   (a) Apply l'Hôpital's Rule twice. What do you observe?

   (b) Factor out x^2 from $x^2 + 1$ in the denominator, cancel, and then evaluate the limit without l'Hôpital's Rule.

   (c) Corroborate your answer to part (b) by plotting and examining the portion of the graph of $y = x/\sqrt{x^2+1}$ for large values of x.

4. Consider $\lim_{x\to 0} \dfrac{\sin x}{x+1}$.

   (a) Use the l'Hôpital's Rule process to obtain a limiting value of the limit.

   (b) Use the limit rules and *not* l'Hôpital's Rule to obtain a limiting value of the limit.

   (c) In a sentence, tell why the result of (a) is wrong.

5. Suppose a right circular cylinder with top and bottom is to be made in such a way that if the radius is r, then the height of the cylinder is 2^{-r}. Determine what happens to the volume V of the cylinder as r increases without bound.

6. EXTRA IF TIME: Solve Exercise 60 in Section 7.6: Show that $\lim_{x\to 0^+} x^{(x^x)} = 0$.