Worksheet for Section 8.2

1. WARMUP: Let $f(x) = \sin x$ for $-\pi/2 \le x \le 0$, and R the region between the graph of f and the x axis on $[-\pi/2, 0]$. Sketch R, and find the volume V of the solid obtained by revolving R about the x axis.

2. (a) Consider the two integrals $\int \sin^8 x \cos^6 x \, dx$ and $\int \sin^7 x \, \cos^4 x \, dx$. Perform the integration for the integral that is easier to integrate.

   (b) Consider the three integrals

   $$\int \tan^6 x \sec^8 x \, dx, \quad \int \tan^3 x \sec^4 x \, dx, \quad \int \tan^4 x \sec^3 x \, dx$$

   Perform the integration for the 2 integrals that are easiest to integrate.

3. Consider the integral $\int \sin^3 x \, dx$.

   (a) Evaluate the integral by an appropriate reduction formula in Section 8.1, and write the constant added at the end as C_1.

   (b) Evaluate the integral by the method of Section 8.2 (i.e., by factoring out $\sin x$ and by writing the rest of the integrand in terms of $\cos x$), and write the constant added at the end as C_2.

   (c) Show that the results found in (a) and (b) are compatible by finding a relationship between C_1 and C_2. (*Hint:* Eliminate $\sin x$ from the answers in (a) and/or (b).)

4. Let r be any positive number (rational or irrational), and consider $\displaystyle\int_0^{\pi/2} \sin^r x \, \cos^3 x \, dx$

   (a) Without evaluating the integral, show that $0 < \displaystyle\int_0^{\pi/2} \sin^r x \cos^3 x \, dx < 2$.

   (b) Evaluate the integral, and then show that its value is less than $\dfrac{2}{3}$, irrespective of the positive value of r.

5. In a short paragraph, describe 3 different kinds of functions you have learned how to integrate in Section 8.2, and the method employed in integrating each of them.