Worksheet for Section 8.3

Warmup:

1. (a) Let $g(t) = \dfrac{t^2}{\sqrt{4-t^2}}$, for $0 < t < 2$. Show that g^{-1} exists.

   (b) Find $\lim\limits_{x\to\infty} x^2\, 5^{-x}$, showing your work.

2. In a sentence, describe the difference between integrating by ordinary substitution (as described in Section 5.6), and integrating by trigonometric substitution.

3. A trigonometric substitution is appropriate for only one of the following integrals, and ordinary substitution (as in Section 5.6) is appropriate for the other two. Evaluate each integral by using an appropriate method, and specify which method you use.

   (a) $\displaystyle\int x\sqrt{x^2-9}\,dx$ (b) $\displaystyle\int \frac{1}{x\sqrt{x^2-9}}\,dx$ (c) $\displaystyle\int \frac{x}{\sqrt{x^2-9}}\,dx$

4. (a) Use integration by parts and trigonometric substitution in order to find $\displaystyle\int x\,\sin^{-1} x\,dx$.

   (b) Find $\displaystyle\int x\,\tan^{-1} x\,dx$.

5. Find the area A of the region between the graph of $f(x) = \dfrac{x^2}{\sqrt{4-x^2}}$ and the x axis on the interval $[-1, 1]$.

6. (a) Use trigonometric substitution to show that if $a > 0$ and $b > 0$, then the region enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ has area $A = \pi ab$.

   (b) If $a = b = 1$, and you want to find the area of the circular region (which is evidently π) by integration, do you need to use trigonometric substitution? Explain why, or why not.