# Worksheet for Section 9.2

1. Find the following limits:

   (a) $\lim_{n\to\infty} \dfrac{(5/n) - 3n^2}{(2/n) - 4n^2}$ (b) $\lim_{k\to\infty} (2k)^{3/k}$ (c) $\lim_{n\to\infty} n \tan\dfrac{\pi}{n}$

2. (a) Let $a_0 = 1$, $a_1 = 1+\frac{1}{3}$, $a_2 = 1+\frac{1}{3}+\frac{1}{9}$, $a_3 = 1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}$, and so on. For an arbitrary positive integer $n$, write down the formula for $a_n$. Then find $\lim_{n\to\infty} a_n$.
   (Hint: Use the formula $1 - r^{n+1} = (1+r+r^2+\cdots+r^n)(1-r)$, for real $r$ and for $n = 1, 2, 3, ...$

   (b) Let $r > 0$, and let $b_n = 1+r+r^2+\cdots+r^n$, for $n \geq 1$. Find $\lim_{n\to\infty} b_n$. (Note that there are two distinct cases, depending on the value of $r$.)

3. You deposit \$100 in a savings account that pays 5% interest compounded annually. Thus after 1 year there is the original \$100 plus the interest $(100)(0.05)$ dollars in the account, that is, there are $100 + (100)(0.05)$ dollars in the account.

   (a) Show that after 2 years there are $100(1 + 0.05)^2$ dollars in the account.

   (b) Let $n$ be an arbitrary positive integer. Find a formula for the amount in the account after $n$ years.

   (c) Determine how many years it would take for the amount in the account to reach \$200.

4. (a) Evaluate $\displaystyle\int_0^1 t^2\, dt$.

   (b) Let $f(t) = t^2$ for $0 \leq t \leq 1$, and let $a_n$ be the right sum for $f$ on $[0, 1]$, where $[0, 1]$ is subdivided into $n$ subintervals of equal length. Find a formula for $a_n$ in terms of $n$.

   (c) Show that $a_n = \dfrac{1^2}{n^3} + \dfrac{2^2}{n^3} + \cdots + \dfrac{n^2}{n^3}$, and find $\lim_{n\to\infty} a_n$.

   (d) Suppose the integral in (a) were $\displaystyle\int_0^2 t^2\, dt$, and that for the right sum the interval $[0, 2]$ were subdivided into $n$ subintervals (as in (b)). What would be a formula for $a_n$?

5. (Extra): Use the ideas of Problem 4 to find $\lim_{n\to\infty} a_n$, where $a_n = \dfrac{1^5}{n^6} + \dfrac{2^5}{n^6} + \cdots + \dfrac{n^5}{n^6}$.