Worksheet for Section 9.6

Warmup:

In the warmup, fill in the blanks with short, reasonable answers. Please fill them out by yourself, and then check your answers with the others in your group.

A. A sequence is a ____________________________, and a series is a ____________________________.

B. The 6th partial sum of $\sum_{n=3}^{\infty} a_n$ is ____.

C. The series $\sum_{n=1}^{\infty} cr^n$ converges and equals ____________________ if and only if ____________________.

D. If $\sum_{n=1}^{\infty} a_n$ diverges, then what we know about $\lim_{n\to\infty} a_n$ is ____________________________________.

---

1. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ is called a p-series. Show that the Ratio Test *cannot* be used to determine the convergence or divergence of the series. (The p-series converges if $p > 1$ and diverges if $0 < p \le 1$.)

2. Let $\displaystyle\sum_{n=m}^{\infty} a_n$ be a positive series. Note that either $\displaystyle\sum_{n=m}^{\infty} a_n$ converges (so partial sums converge to a limit), or the partial sums s_j approach ∞ as j grows without bound (and we associate $\displaystyle\sum_{n=m}^{\infty} a_n$ with ∞).

   COMPARISON TEST: Let $\displaystyle\sum_{n=m}^{\infty} a_n$ and $\displaystyle\sum_{n=m}^{\infty} b_n$ be positive series.

   i. If $\displaystyle\sum_{n=m}^{\infty} a_n$ converges and $0 \le b_n \le a_n$ for all n, then $\displaystyle\sum_{n=m}^{\infty} b_n$ converges.

   ii. If $\displaystyle\sum_{n=m}^{\infty} a_n$ diverges and $0 < a_n \le b_n$ for all n, then $\displaystyle\sum_{n=m}^{\infty} b_n$ diverges.

   Use the Comparison Test, and the Ratio Test or Root Test where applicable, to determine whether each of the following series converges, or diverges.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)(n+1)}$ (Hint: Use Exercise 1 above.)

3. Use the Ratio Test to determine if the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (n!)^2}{(2n)!}$ converges.

4. Let $a_{2n} = 1/2^n$ and $a_{2n+1} = 1/3^n$ for all $n \ge 1$. Show that $\lim_{n\to\infty}(a_{n+1}/a_n)$ does not exist, but nevertheless by the Comparison Test $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.