1.   a(10). A student uses identical paper clips to measure the length of a desk. He finds that the desk is 42 paper clips long. Another student used identical straws to measure the length of a desk. Two straws are the same length as seven paper clips. How many straws long is the desk? Explain your reasoning.

$$\frac{7 \text{ clips}}{2 \text{ straws}} \xrightarrow{\times 6} = \frac{42 \text{ clips}}{x \text{ straws}} \xrightarrow{\times 6} \rightarrow x = 12$$

There are 6 groups of 7 paperclips in the desk length. $(6 \times 7 = 42)$
So there are 6 groups of 2 straws in the desk length → 12 straws total.

b(15). Usain Bolt is the world's fastest person, holding the current world record in the 150 meter race. In 2015, he earned approximately $20 million. On 27 August 2015, Bolt won the final of the 200 meters with a time of 19.55 seconds at the 2015 Beijing World Champions. How many miles per hour was he running during this race? Use dimensional analysis and the equivalence 1 inch = 2.54 cm. Show your work. Round your **final** answer to the nearest tenth. (Do not round until the very end!)

Bolt celebrating at the 2013 London Anniversary Games.

$$\frac{200 \text{ meters}}{19.55 \text{ seconds}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ mile}}{5280 \text{ ft}}$$

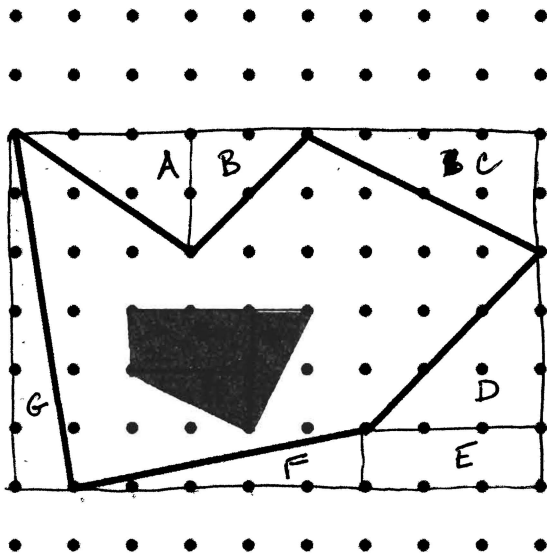$$\rightarrow \approx .1242742384 \text{ miles in } 19.55 \text{ seconds}$$

$$\frac{.1242742384 \text{ miles}}{19.55 \text{ sec's}} \times \frac{60 \text{ sec's}}{1 \text{ min}} \times \frac{60 \text{ min}}{1 \text{ hr}}$$

$$\rightarrow \approx 22.9 \text{ miles per hour.}$$

2(10). Below is the outline of Madai's new yard. The scale is 1 square = 1 square yard. She is going to make the yard into a brick patio, with space for a pool (shaded portion). What is the area of her patio? Find the exact area using basic area concepts (# square units) and strategies such as subdivide or surround-and-subtract. You may assume that the diagonal of a rectangle cuts the area in half. Show all work. Annotate the diagram so your thinking process is clear.

Total area outlined (rectangle) is
$6 \times 9 = 54$

Areas to subtract:
A = ½ of 3×2 = 3
B = ½ of 2×2 = 2
C = ½ of 4×2 = 4
D = ½ of 3×3 = 4½ } 22
E = 3
F = ½ of 1×5 = 2½
G = ½ of 1×6 = 3

55 − 22 = 33 sq yards patio + pool

Subtract pool area: 33 − 4 = 29 sq yds //

3(9). Joshua plants peas and carrots in his garden in the spaces shown below right.

Complete each sentence below with an appropriate fraction to make true statements:

Joshua planted peas in ___¼___ square yards of his garden.

Joshua planted carrots in ___3/16___ square yards of his garden.

The area he planted with peas is ___4/3___ as big as the area he planted with carrots.

1 yard

Pea

Carrot

2 yards
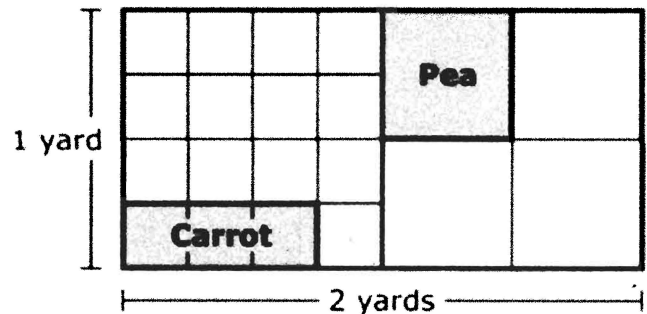
4. One of the seven wonders of the world is the Great Pyramid of Cheops or Khufu in Giza, Egypt. It is the only pyramid still standing at this location.

The diagram at right gives its dimensions in Royal Cubits. Assume the base is a square and that the triangles are all congruent. Assume the dimension of 280 cubits is from the apex and perpendicular to the center of the square.
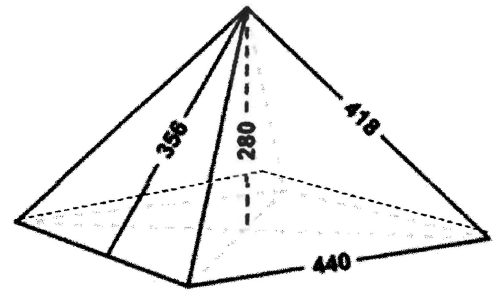
a(12). Find the surface area of the pyramid using Royal Cubits. Show all work.

Each triangle area = $\frac{1}{2}(440)(356) = 78320$

Four triangles: $4 \times 78320 = 313,280$

Base: $440^2 = 193,600$

Total SA = 506,880 square cubits

b(8). Convert the surface area you found in part a to square feet (Note: 1 Royal Cubit = 20.6 inches). Show all work.

$$506,880 \ c^2 \times \frac{(20.6)^2 \ in^2}{1 \ c^2} \times \frac{1 \ ft^2}{12^2 \ in^2} = 1,493,747.2 \ ft^2$$

Extra Credit (+1): Jose thinks the surface area of the Great Pyramid is probably about ten times the surface area of the Washington Monument. The Washington Monument's surface area is approximately 93,587 sq ft. How close is Jose to being right?

5(10). Using the trapezoid diagram below, show using strategies such as cut-and-move, surround-and-subtract, or double-and-take-half that the formula **Area of Trapezoid** $= \frac{1}{2}(B + b)h$ is sensible. Explain briefly how your sketches and process justify the formula.

b

B

h

Second copy of trapezoid, rotated and put together with original

B

b
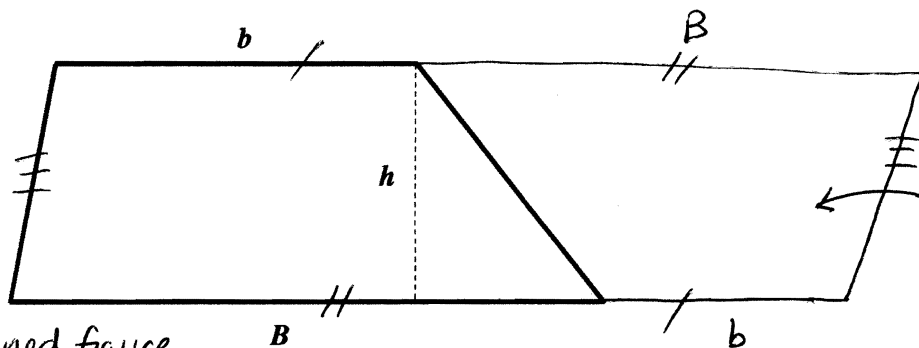
In combined figure,
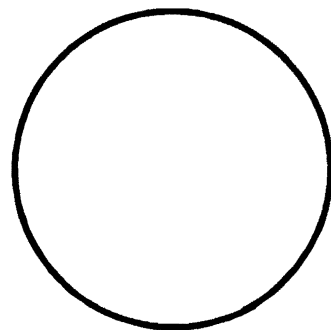Both pairs of opposite sides are $\cong$, so it is a p'gram.

Large parallelogram has base B+b and height h
So its area is $(B+b)\cdot h$.

The trapezoid is half of this area, so $\frac{1}{2}(B+b)h$.

6(8). What is the circumference of a circle whose area is 25π? Show work. Give your final answer in terms of π.

$$A = \pi r^2 = 25\pi \rightarrow r = 5$$
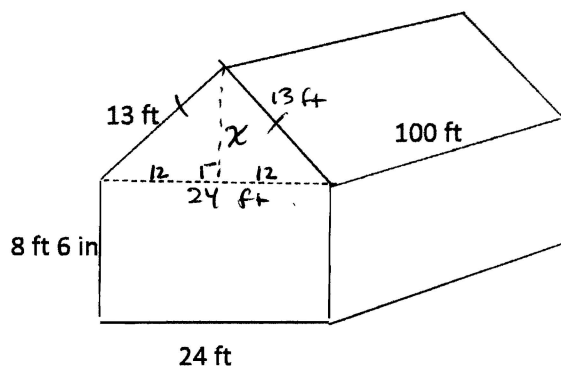
$$C = 2\pi r = 2\pi(5) = 10\pi$$

7(18). Consider the barn in the shape of a prism shown at right. The pentagon face is in the shape of a rectangle topped with an isosceles triangle.

Find the volume of the prism. Show all work. Include appropriate units on your final answer.

$V = A h$ ← distance between pentagons

↑ area of pentagon

Find x = height of triangle:

$12^2 + x^2 = 13^2 \rightarrow x = 5$

Area of triangle: $\frac{1}{2}(24)(5) = 60 \ ft^2$ ⎫ Pentagon area
Area of rectangle: $(24)(8.5) = 204 \ ft^2$ ⎬ $= 264 \ ft^2$
⎭

$Volume = (264)(100) = 26{,}400 \ ft^3$

13 ft 13 ft x 100 ft
12 5 12
24 ft
8 ft 6 in
24 ft

Extra Credit(up to +3): The diagram shown at right is a pyramid with a triangular base where the three triangles meeting at point A all have a right angle at A. (Imagine A is the corner of a box; the sides of the box have been cut to form the triangles shown, and a new piece of cardboard covers the slanted (shaded) triangle shown.)

Find the surface area of this shape (include all four triangular faces). Show all work.

2 m
2 m
A
2 m
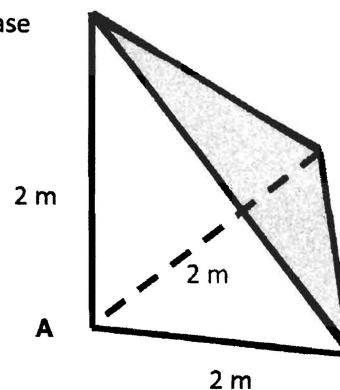
Please copy and sign: I pledge on my honor that I have not given or received any unauthorized assistance on this exam. [signed]