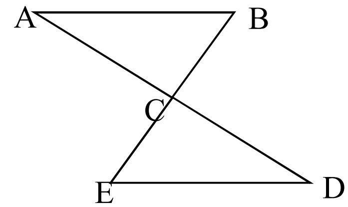
Thus, our final write-up is as follows:

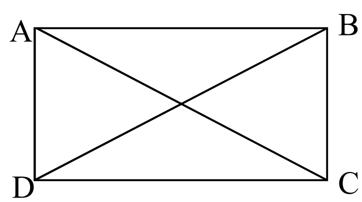


Statement	Reason
1. $\overline{AB} \parallel \overline{DE}$ $\overline{BC} \cong \overline{EC}$	1. given
2. $\angle ACB \cong \angle DCE$	2. Vertical angles are congruent
3. $\angle ABC \cong \angle DEC$	3. If parallel lines, alternate interior angles are congruent
4. $\Delta ABC \cong \Delta DEC$	4. ASA

Example 4

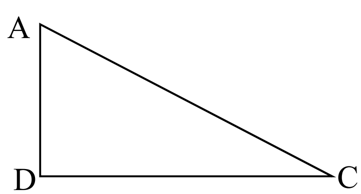
Given: $\overline{AD} \cong \overline{BC}$
$\overline{AD} \perp \overline{DC}$, $\overline{BC} \perp \overline{DC}$

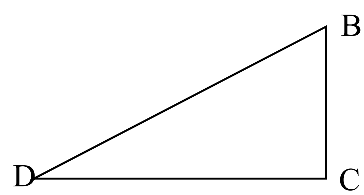
Prove: $\Delta ADC \cong \Delta BCD$



Solution:

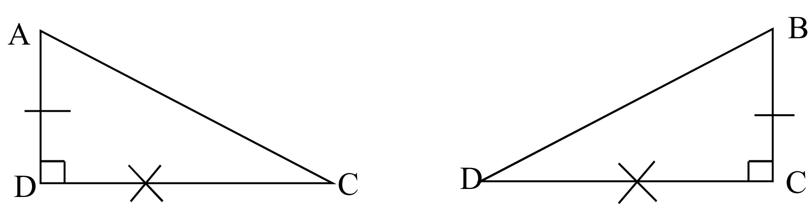
This proof looks a little more difficult than it is because the two triangles we want to prove congruent overlap. It is helpful to redraw the shapes separately, as shown:





Now we can begin marking the congruences we are given and can derive on our redrawn diagram. $\overline{AD}$ and $\overline{BC}$ are given to be congruent. The perpendicular segments form right angles, by definition of perpendicular; of course, all right angles are congruent (Euclid took this as a postulate).

We have now "used up" all of the stated "Given," yet we still need one more pair of congruent sides or angles. At this point, it is advisable to look back at the original, overlapping, diagram. Any shared side or angle will be congruent to itself by the reflexive property. In this case, the two triangles we want to prove congruent both have $\overline{DC}$ as one side. Marking an "X" on each of the separated triangles, we obtain the figure below:



Finally, we can see that these triangles are congruent by the SAS property, and we are ready to write the formal proof.

Statement	Reason
1. $\overline{AD} \cong \overline{BC}$ $\overline{AD} \perp \overline{DC}, \overline{BC} \perp \overline{DC}$	1. given
2. ∠ADC, ∠BCD are right angles	2. Def'n of perpendicular
3. ∠ADC ≅ ∠BCD	3. All right angles are congruent
4. $\overline{DC} \cong \overline{DC}$	4. reflexive property
5. ΔADC ≅ ΔBCD	5. SAS

Online Practice Examples

A couple online practice examples are available at the links below. The numbers given are for proofs similar to those in this section. Try to write the proof yourself before checking the solution.

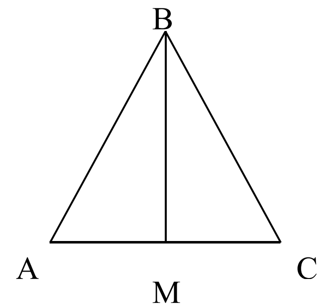
https://mathbitsnotebook.com/Geometry/CongruentTriangles/CTbeginning.html #1, 2, 3

https://mathbitsnotebook.com/Geometry/CongruentTriangles/CTproofs.html #2

Practice Proofs:

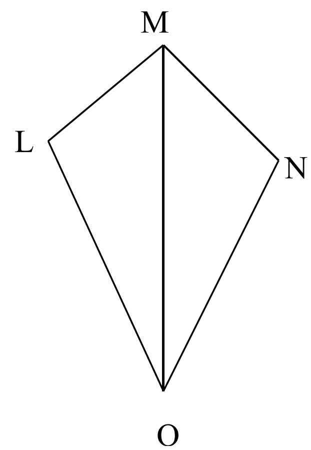
#1. Given: $\overline{BM} \perp \overline{AC}$, M is the midpoint of $\overline{AC}$
Prove: $\Delta ABM \cong \Delta CBM$

#2. Given: $\overline{AB} \cong \overline{BC}$; $\overline{BM}$ bisects $\overline{AC}$.
Prove: $\Delta ABM \cong \Delta CBM$



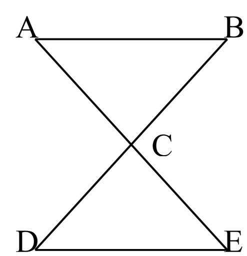
#3. Given: $\overline{MO}$ bisects $\angle LMN$ and $\angle LON$
Prove: $\Delta MLO \cong \Delta MNO$

#4. Draw $\overline{LN}$, intersecting $\overline{MO}$ at point P.
Given: $\overline{MO}$ is the perpendicular bisector of $\overline{LN}$.
Prove: $\Delta MLP \cong \Delta MNP$

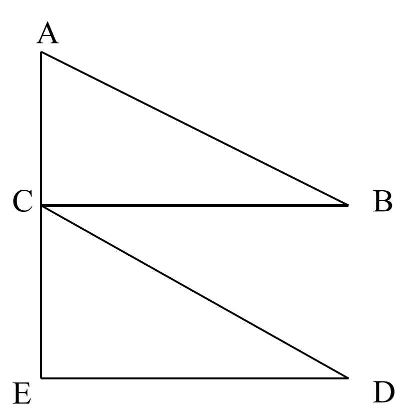


#5. Given: $\overline{AB} \parallel \overline{DE}$; $\overline{AB} \cong \overline{DE}$
Prove: $\Delta ABC \cong \Delta EDC$

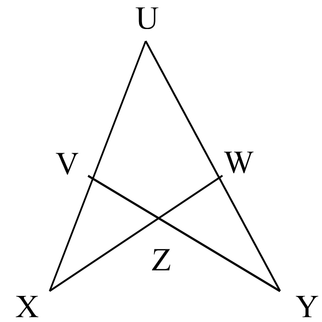
#6. Given: $\overline{AE}$ and $\overline{BD}$ bisect each other at C.
Prove: $\Delta ABC \cong \Delta EDC$



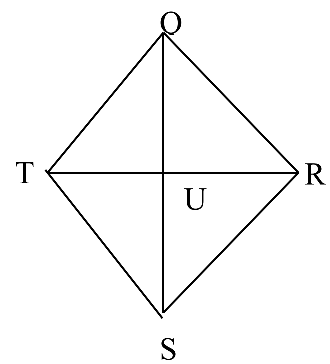
#7. Given: $\overline{AB} \parallel \overline{CD}$; $\overline{BC} \parallel \overline{DE}$
C is the midpoint of $\overline{AE}$
Prove: $\Delta ABC \cong \Delta CDE$



#8. Given: $\overline{UX} \cong \overline{UY}$; $\angle UXZ \cong \angle UYZ$
Prove: $\Delta UXW \cong \Delta UYV$



#9. Given: QRST is a rhombus; $\overline{QS} \cong \overline{RT}$
Prove: $\Delta QRS \cong \Delta RST$



#10. Prove the AAS Triangle Congruence Property.

#11. Prove: If two angles are supplementary and congruent they are right angles.