

Doubling a Point on an Elliptic Curve

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curve:=(y^2=x^3+x+6)
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$$y^2 = x^3 + x + 6$$

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tanline:=(y-b=((3*a^2+1)/(2*b))*(x-a))
```

$$y - b = -\frac{(3a^2 + 1)(a - x)}{2b}$$

```
tanline1:=subs(tanline,a=3,b=6)
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$$y - 6 = \frac{7x}{3} - 7$$

```
solve([curve,tanline1],[x,y])
```

$$\left\{ [x = 3, y = 6], [x = -\frac{5}{9}, y = -\frac{62}{27}] \right\}$$

```
tanline1:=subs(tanline,a=-5/9,b=62/27)
```

$$y - \frac{62}{27} = \frac{13x}{31} + \frac{65}{279}$$

```
solve([curve,tanline1],[x,y])
```

$$\left\{ [x = -\frac{5}{9}, y = \frac{62}{27}], [x = \frac{11131}{8649}, y = \frac{2468546}{804357}] \right\}$$

```
tanline1:=subs(tanline,a=11131/8649,b=-2468546/804357)
```

$$y + \frac{2468546}{804357} = \frac{414168447967}{330932042487} - \frac{37208557x}{38262463}$$

```
solve([curve,tanline1],[x,y])
```

$$\left\{ [x = \frac{11131}{8649}, y = -\frac{2468546}{804357}], [x = -\frac{21454304639132357}{13176144673437321}, y = -\frac{353956079228907772869506}{1512455244150127732744869}] \right\}$$