MATH 608K (Algebraic K-Theory)
Homework Assignment #3, Fall, 2005
Group Homology, Central Extensions, and K_2

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due Wednesday, November 30, 2005

1. Do Exercise 4.1.31 in the book, computing $H_\bullet(F, \mathbb{Z})$ and showing that it is isomorphic to $\bigwedge^\bullet F$, when F is a free abelian group. If you can't do the general case, at least do the cases where F is of rank 1 and rank 2. The latter gives the isomorphism $H_2(\mathbb{Z}^2, \mathbb{Z}) \cong \mathbb{Z}$ which we mentioned in class.

2. Do Exercise 4.1.28 in the book on H_2 for $SL(2, \mathbb{F}_q)$ and $SL(3, \mathbb{F}_q)$ for some small finite fields $\mathbb{F}_q$. (In part (4), to show $SL(2, \mathbb{F}_5)$ is isomorphic to the universal central extension of A_5, observe that the center of $SL(2, \mathbb{F}_5)$ is cyclic of order 2, and show that the quotient by the center, $PSL(2, \mathbb{F}_5)$, is a perfect group of order 60. But it is known there is only one such group, A_5. It is an interesting exercise to construct an explicit isomorphism $PSL(2, \mathbb{F}_5) \to A_5$.)

3. If G_1 and G_2 are perfect groups with universal central extensions E_1 and E_2, check that $E_1 \times E_2$ is a universal central extension of $G_1 \times G_2$. Deduce that $H_2(G_1 \times G_2, \mathbb{Z}) \cong H_2(G_1, \mathbb{Z}) \oplus H_2(G_2, \mathbb{Z})$. Use this to do Exercise 4.3.18, in other words, to show that K_2 of a direct product of rings is the direct product (or sum, depending on whether you use multiplicative or additive notation) of the K_2 groups.