MATH 241
MATLAB Problem Set 4
Line Integrals
Due Thursday, December 6

You will need to download the two MATLAB Mfiles `lint.m, curl.m` from the course web page. Both of these were written by Prof. Jeffrey Cooper. To see how to use `lint.m`, enter `type lint` at the MATLAB prompt. The directions for use are at the beginning of the file. The same applies for `curl.m`.

1. Seeing a 2-dimensional vector field. Make an Mfile that will display an arbitrary vector field in two dimensions. If you want, you can simply cut and paste the first few (non-comment) lines of lint.m Using your Mfile, display the vector field $\mathbf{F}(x, y) = y^2\,\mathbf{i} + x/10\mathbf{j}$ in the rectangle $0 \le x \le 2$, $1 \le y \le 2$. Note: When inputting the funtions u and v, it is simplest to use the anonymous function (i.e., @) format. Be sure to make your functions 'vector-smart' using dots.

2. Let $\mathbf{F}(x, y) = y^2\,\mathbf{i} + x/10\mathbf{j}$ be the same two-dimesional vector field as in (1). Use the Mfile `lint.m` to do the following calculations on the rectangle $R = \{0 \le x \le 1, 1 \le y \le 2\}$:

a) Use just one segment C from $P = (.2, 1.2)$ to $Q = (.8, 1.8)$. What is the value of the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$?

b) Use just one segment C' from $P' = (.8, 1.2)$ to $Q' = (.2, 1.8)$. What is the value of the line integral this time? How do you explain the difference in signs between the result of part a) and that of part b)? Look at the angle between the vector $\mathbf{F}$ and the tangent vectors to the segments.

c) Now use four segments, tracing out roughly a square with the vertices taken in this order P, P', Q', Q and back to P. On which segment is the integral positive? On which is is it negative? On which segments is it very small? Explain.

3. Let the vector field be $\mathbf{F} = \sin(xy)\,\mathbf{i} + (x - y)\,\mathbf{j}$ and let the square $R = \{0 \le x \le 2.5, 0 \le y \le 2.5\}$. Set the vector `corners = [0 2.5 0 2.5]`.

a) Let C be the path consisting of the two straight segments from $P = (.5, .5)$ to $(1.5, 1)$, and from $(1.5, 1)$ to $Q = (2, 2)$. Use `lint` to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.

b) Repeat the calculation with a different path C' from P to Q. Use any number of segments. Are the results from part a) and b) the same? Is this vector field conservative?

c) Calculate the line integral in the counterclockwise direction around a small

triangle, that is centered at the point $(2, 2)$. Then from the information displayed on the screen, calculate the ratio (total line integral)/area. Compare the ratio you calculated with the value of the curl (calculated by hand) at the point $(2, 2)$. To make the values closer, use a smaller triangle.

d) Use the command `curl(u,v,corners)` to get a color map of the curl of $\mathbf{F}$.

3. Let $\mathbf{F}(x, y) = x + \sin y\,\mathbf{i} + x\cos y\,\mathbf{j}$. Let the rectangle $R = \{1 \le x \le 4, -2 \le y \le 2\}$.

a) Use the Mfile `lint.m` to calculate the line integral along any two paths from the point $P = (1.5, -1)$ to the point $Q = (3.5, 1)$. Use any number of segments.

b) Calculate the line integral around any closed path. What can you conclude about this vector field? Verify your conclusion with a hand calculation.