MATH 241 (Laskowski) Fall, 2007
MATLAB Problem Set 2
Due Tuesday, October 23

Note: Matlab online help commands gives examples of `meshgrid` and `surf`.

1. Enter the function
$$f(x,y) = \cos(x^2 + 4y^2).$$
Be sure to make it "array smart" (or "vectorized"). i.e., use dots.

a) Graph f over the square $D = \{|x| \leq 2, |y| \leq 2\}$. Hint: Use the `meshgrid` command to make the set of grid points and then use `surf`. Label the axes.

b) Plot the level curves (contours) of f in the xy plane in the same square D. You can use the command `contour` (works analogously to `surf`). Label the axes.

Note: You can put the plots of parts a) and b) together on the same page with the command `subplot`.

2. Enter the function
$$u(x,y) = (-4x^3 + 3x^2 + 1)(y - y^2).$$
Make sure it is "array smart". The function u is the temperature at a point (x, y) in the unit square $Q = \{0 \leq x \leq 1,\ 0 \leq y \leq 1\}$. The heat flux at each point is defined to be the *negative* of the gradient vector $-\nabla u(x,y) = -[u_x(x,y), u_y(x,y)]$.

a) Verify by hand that $u_x(0, y) = 0$ for $0 \leq y \leq 1$ and that $u = 0$ on the other three edges of the square. (*** You do not need to turn this part in.) This means that the left edge of the square is insulated, and that the temperature is held at zero on the other three edges.

b) Put a 20 by 20 meshgrid on Q. Graph u over Q using the command `surf(X,Y,u(X,Y))`. Note where the temperature is greatest, and the appearance of the surface on the edge $x = 0$.

c) Compute the partial derivatives u_x and u_y. You can either have MATLAB do this, or you can compute the partial derivatives by hand and then enter the functions `ux` and `uy` into MATLAB by hand. Then enter the commands

```
U  = u(X,Y);
Ux = ux(X,Y);
Uy = uy(X,Y);
contour(X,Y,U,20)
hold on
quiver(X,Y,-Ux,-Uy)
```

Note: The last command attaches arrows at each point of the meshgrid to represent the vector field of the heat flux.

d) Use the plot of part c) to answer the following questions.

- What is the direction of the heat flux with respect to the level curves?
- Where is the 'hot spot', and which way is the heat flowing?
- What is the angle at which the level curves meet the edge $x = 0$?
- What is the direction of the heat flux at that edge of the square? How do you explain this physically?
- Why are the flux vectors perpendicular to the other edges?

3. Let $f(x, y) = x^4 - 2x^2 - y^3 + 3y$. This function has six critical points in the square $D = \{|x| \leq 2, |y| \leq 2\}$.

a) Make a fine mesh over the square D, say 50 by 50. Then use the command `contour(X,Y,f(X,Y),levels)` where `levels = linspace(-4,4, 21)`. Look at the contour map to locate the six critical points. On the printout of the contour map, indicate the location of each critical point, and indicate its nature.

b) Now solve the system $f_x = 0,\ f_y = 0$ (either by hand or using MATLAB). Use the second derivative test at each critical point. Compare these results with your observations in part a).