MATH 241 (Laskowski) Fall, 2007
MATLAB Problem Set 3
Due Thursday, November 8

There are two ways of 'computing integrals' namely symbolically or numerically. Symbolic integration is more straightforward.

1. (Symbolic integration) Compute $\iint_R f\,dA$ symbolically, where $f = x^2 + y$ and R is the region in the xy-plane bounded below by the x-axis, bounded above by the equation $y = x^2$, between $x = 1$ and $x = 3$.

(a) Create an mfile with:

```
syms x y;
f = x^2 + y;
int(int(f,y,0,x^2),1,3)
```

Notes: f is NOT vectorized as we are working symbolically! Also, the second statement could be replaced by `int(int(f,y,0,x^2),x,1,3)`

(b) Run the mfile to get the answer.

2. (a) Have MATLAB do #24 of Section 14.1.

(b) Can MATLAB do 'reversing the integral' problems such as #57 of 14.1 ? Try it.

3. (Numerical double integration) The basic commands are `dblquad` and `triplequad`. MATLAB7 has built in documentation. `dblquad` is rather straightforward when one is integrating over a rectangular region. However, when using either of these commands, the function you are integrating must be vectorized.

(a) Type in (or create an mfile and run the mfile) for

```
f=@(x,y) cos(x.^2.*y);  Note: vectorized!
dblquad(f,0,1,0,2)
```

This gives a numerical estimate for the double integral of f over the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 2$.

There are many times when you might want to change between one representation of a function and another. For example, if you want MATLAB to compute derivatives or do symbolic integration, you want to represent functions symbolically, In general, computational methods require functions to be vectorized. The following example shows how to change a function from symbolic form to the '@' form:

```
syms x y;
ff=cos(x^2 * y); (Symbolic form!)
g=sqrt(diff(ff,x)^2 + diff(ff,y)^2 + 1); (Useful for surface areas)
h=@(x,y) eval(vectorize(g)) Then h is ready to plug into dblquad
```

(b) Using the ideas above and in Section 14.3, estimate the surface area of the portion of the surface $z = x^2y$ lying over the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 2$.

4. `dblquad` over non-rectangular regions.

Since `dblquad` is simply giving a numerical estimate (as opposed to solving the iterated integrals directly) one needs to be clever to handle non-rectangular regions. The key is to use 'logical functions.' We begin with an example.

```
g=@(x,y) y< 2.*x
```

Now $g(x, y) = 1$ whenever $y < 2x$ and $g(x, y) = 0$ whenever $y \geq 2x$.

(a) Test this out by computing `g(3,5)`, `g(2,7)`, and `g(3,6)`.

So if we let $f(x, y) = x^2y$ and define h by `h=@(x,y) f(x,y).*g(x,y)` then $h(x, y) = x^2y$ whenever $y < 2x$ and $h(x, y) = 0$ whenever $y \geq 2x$. Thus,

$$\int_0^1 \int_0^{2x} f(x,y)\,dy\,dx \quad = \quad \int_0^1 \int_0^2 h(x,y)\,dy\,dx$$

and we know how to use estimate double integrals over rectangular regions from above.

(b) Estimate the surface area of the portion of the surface $z = x^2y$ lying over the interior of the triangle in the xy-plane with vertices at $(0, 0)$, $(1, 2)$, $(1, 0)$.

(c) Note that the triangle mentioned above has exactly half of the area of the rectangle described in 3(b). Is the answer to 4(b) exactly half of the answer in 3(b) ? If it is, explain why it is equal. If it is not, explain why it is not.

(d) Compute the surface area of the portion of the graph of $z = exp(x^2y)$ lying over the 'hump' on the xy-plane lying above the x-axis, below the graph of $y = \sin(x)$, between $x = 0$ and $x = \pi$.