MATH 141H Sample Exam 3 Solutions

1. Ratio Test:

$$\frac{a_{n+1}}{a_n} = \frac{(2n+2)!2^n(n!)^2}{2^{n+1}((n+1)!)^2(2n)!} = \frac{(2n+2)(2n+1)}{2(n+1)^2} \to 2 = r > 1$$

as $n \to \infty$. Therefore, the series diverges.

2.

$$R = \lim_{n\to\infty} \frac{1/\sqrt{n}}{1/\sqrt{n+1}} = 1.$$

Therefore, the series converges for $|x| < 1$ and diverges for $|x| > 1$. Now check the endpoints: when $x = 1$, we get $\sum 1/\sqrt{n}$, which diverges. When $x = -1$ we get $\sum(-1)^n/\sqrt{n}$, which converges by the alternating signs test. Final answer: the series converges for $-1 \leq x < 1$.

3. We know $e^t = 1 + t + t^2/2! + t^3/3! + \cdots$. Substitute $t = x^3$ and multiply the result by x^2 to get

$$x^2 e^{x^3} = x^2 + x^5 + \frac{x^8}{2!} + \frac{x^{11}}{3!} + \cdots .$$

4. $\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \cdots$. Therefore

$$\cos(.1) = 1 - \frac{.01}{2!} + \frac{.0001}{4!} - \cdots .$$

This has alternating signs and decreasing terms, so we can estimate the error by the first omitted term. If we stop after two terms, the first omitted term is $.0001/4! < .0001$, which is what we want. Therefore,

$$\cos(.1) = 1 - \frac{.01}{2} = .995$$

with an error less than .0001.

5. (a) This has alternating signs and (except for the first term) terms decreasing to 0, so it converges.

(b) this is the Taylor series for $\sin \pi$, which equals 0.

6. (a) $4e^{i\pi/3} = 4(\cos(\pi/3) + i\sin(\pi/3)) = 2 + 2i\sqrt{3}$

(b) Multiply numerator and denominator by $(4 - 5i)$ to obtain

$$\frac{(3+2i)(4-5i)}{(4+5i)(4-5i)} = \frac{22-7i}{41} = \frac{22}{41} - i\frac{7}{41}$$

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