MATH 141H Exam 2 Nov. 11, 2002

1. (a) (8 points) For what values of r does $\displaystyle\int_0^1 \frac{dx}{(1-x)^r}$ converge? Justify your answer.

(b) (8 points) For what values of r does $\displaystyle\int_1^\infty \frac{dx}{x^r}$ converge? Justify your answer.

2. (12 points) Evaluate $\lim_{n\to\infty}\left(\dfrac{n}{n+1}\right)^{n-1}$.

3. (12 points) Take the region between the curves $y=\pm\sqrt{R^2-x^2}$ such that $|y|\le 3$ (note that this is an inequality for y, not x) and revolve it around the y-axis. What is the volume?

Another way to look at this is as follows: You have a solid sphere of radius R and you drill a cylindrical hole through its center such that the sides of the hole have length 6. What is the volume of the remaining part of the sphere?

4. (12 points) Evaluate $\displaystyle\int \frac{2x^2+3}{x(x-1)^2}\,dx$.

5. (12 points) Evaluate $\displaystyle\int \frac{2x}{(x+1)(x^2+1)^2}\,dx$.

6. (12 points) Find the surface area when the curve $y=x^3,\quad 0\le x\le 2$ is revolved around the x-axis.

7. (12 points) A pyramid is such that the cross-section at height x is a $30-x$ by $30-x$ square, for $0\le x\le 30$. Express the volume of the pyramid as an integral and determine the volume.

8. (12 points) A ball is dropped from a height of 6 feet onto a smooth surface. On each bounce, the ball rises to 3/4 of the height it reached on the previous bounce. Find the total distance the balls travels.

9. (30 points) For each of the following series, determine whether it converges or diverges. Explain how you obtain your answer.

$$(a)\quad \sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+n+3} \qquad (b)\quad \sum_{n=2}^{\infty}\frac{(\ln n)^2}{n} \qquad (c)\quad \sum_{n=2}^{\infty}\frac{1}{(\ln n)^2 n}$$

10. (15 points) Show that

$$\left|\sum_{n=1001}^{\infty}\frac{1}{n^2}\right| < .001$$

Useful formulas:

$$\pi \int_a^b f(x)^2 \, dx, \quad 2\pi \int_a^b x f(x) \, dx$$

$$\int_a^b \sqrt{1 + f'(x)^2} \, dx, \quad 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} \, dx$$