Math 246, First In-Class Exam (Wednesday, 5 March, 2003)

Professor Levermore

Before doing anything else, on the cover of your examination booklet please print your name and student identification number. Then print the UMD Honor Pledge and sign your name under it:

I pledge on my honor that I have not given or received
any unauthorized assistance on this examination.

Exam ends at 5 minutes of the hour, no later. Exam is closed book and closed notes. No calculators or other electronic devices are allowed. Work at most one problem on each page of your examination booklet, marking the problem number at the top of the page. Your answer(s) to each part should be circled, and you must show your work. Brief descriptions of your reasoning are helpful, especially for earning partial credit. Any work that you do not want to be considered should be crossed out.

(1) (12 points) Consider the differential equation

$$\frac{dy}{dt} = -y^2(y-2)(y-4)\,.$$

  (a) Find all of the equilibrium (stationary) solutions and classify each as stable, unstable, or semistable.

  (b) Draw a graph of y versus t showing the direction field and several solution curves, including all of the equilibrium solutions and solutions above and below each equilibrium value.

  (c) If $y(0) = 3$, what is the limiting value of y as $t \to \infty$?

(2) (18 points) Give an explicit solution to each of the following initial value problems. Identify the interval over which each solution is defined.

  (a) $\dfrac{du}{dz} = \dfrac{\cos(z)}{1+u}\,,\quad u(0) = 0\,.$

  (b) $\dfrac{dz}{dt} = \dfrac{e^t - z}{2+t}\,,\quad z(0) = 3\,.$

(3) (18 points) Suppose a room containing 50 m^3 (m = meters) of air is initially free of carbon monoxide. Beginning at $t = 0$ cigarette containing 4% carbon monoxide is introduced into the room at a rate of .001 m^3/min, and the well-circulated mixture is allowed to leave the room at the same rate. Let $C(t)$ denote the volume of carbon monoxide in the room at time $t \geq 0$.

  (a) Is $C(t)$ an increasing or decreasing function of t? (Give reasoning.)

  (b) What is the behavior of $C(t)$ as $t \to \infty$? (Give reasoning.)

  (c) Derive a formula for $C(t)$.

(4) (6 points) Suppose you are using the improved Euler method to numerically solve an initial-value problem over the interval [0,20]. By what factor would you expect the error to decrease when you increase the number of steps taken from 500 to 2000?

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(5) (18 points) Give an implicit general solution to each of the following differential equations.

(a) $(3x^2 - 2xy)\, dx + (6y^2 - x^2)\, dy = 0$.

(b) $\left(\dfrac{y}{x} + 3x\right) dx + dy = 0$.

(6) (16 points) A 9 kilogram (kg) mass initially at rest is dropped in a medium that offers a resistance force of $v^2/20$ newtons ($= \text{kg m/sec}^2$) where v is the downward velocity of the mass in meters per second. The gravitational acceleration is 9.8 m/sec^2.

(a) What is the terminal velocity of the mass?

(b) Write down an initial value problem that governs v as a function of time. (You do not have to solve it!)

(7) (12 points) Consider the following MATLAB function M-file.
function [t,y] = solveit(ti, yi, tf, n)

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h = (tf - ti)/n;
t = zeros(n + 1, 1);
y = zeros(n + 1, 1);
t(1) = ti;
y(1) = yi;
for i = 1:n
t(i + 1) = t(i) + h;
y(i + 1) = y(i) + h*(y(i) - y(i)^3);
end
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(a) What is the initial-value problem being solved numerically?

(b) What is the numerical method being used to solve it?

(c) What are the output values of t(2) and y(2) that you would expect for input values of ti = 0, yi = .5, tf = 5, n = 20 for this method?