Math 246, Sample Problem Solutions for Second In-Class Exam

(1) Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (shown with multiplicities) are $-2+3i$, $-2-3i$, $7i$, $7i$, $-7i$, $-7i$, 5, 5, -3, 0, 0, 0.

(a) What is the order of L?

Solution: There are twelve roots listed, so the degree of the characteristic polynomial is twelve, and consequently the order of L must be twelve.

(b) Give a general real solution of the homogeneous equation $Ly = 0$?

Solution: The general solution is

$$\begin{aligned} y = c_1 e^{-2t}\cos(3t) &+ c_2 e^{-2t}\sin(3t) \\ &+ c_3\cos(7t) + c_4\sin(7t) + c_5 t\cos(7t) + c_6 t\sin(7t) \\ &+ c_7 e^{5t} + c_8 t e^{5t} + c_9 e^{-3t} \\ &+ c_{10} + c_{11} t + c_{12} t^2 \,. \end{aligned}$$

The reasoning is as follows.

- The conjugate pair $-2 \pm 3i$ yields $e^{-2t}\cos(3t)$ and $e^{-2t}\sin(3t)$.
- The double conjugate pair $\pm 7i$ yields

$$\cos(7t)\,, \quad \sin(7t)\,, \quad t\cos(7t)\,, \quad \text{and} \quad t\sin(7t)\,.$$

- The double real root 5 yields e^{5t} and $t\,e^{5t}$.
- The simple real root -3 yields e^{-3t}.
- The triple real root 0 yields 1, t, and t^2.

(2) Solve each of the following initial-value problems.

(a) $y'' + 4y' + 4y = 0\,, \quad y(0) = 1\,, \quad y'(0) = 0\,.$

Solution: This is a constant coefficient, homogeneous linear problem. Its characteristic polynomial is

$$P(z) = z^2 + 4z + 4 = (z+2)^2\,.$$

It has the double real root -2, which yields the general solution

$$y(t) = c_1 e^{-2t} + c_2 t\,e^{-2t}\,.$$

Because

$$y'(t) = -2c_1 e^{-2t} + c_2\left(e^{-2t} - 2t\,e^{-2t}\right),$$

when the initial conditions are imposed, one finds that

$$y(0) = c_1 = 1\,, \qquad y'(0) = -2c_1 + c_2 = 0\,.$$

These are solved to find that $c_1 = 1$ and $c_2 = 2$. The solution of the initial-value problem is therefore

$$y(t) = e^{-2t} + 2t\,e^{-2t} = (1+2t)\,e^{-2t}\,.$$

(b) $y'' + y = 4e^t\,, \quad y(0) = 0\,, \quad y'(0) = 0\,.$

Solution: This is a constant coefficient, inhomogeneous linear problem. The characteristic polynomial of its homogeneous part is

$$P(z) = z^2 + 1\,.$$

It has the conjugate pair of roots $\pm i$, which yields the general homogeneous solution

$$y_H(t) = c_1 \cos(t) + c_2 \sin(t)\,.$$

Because the forcing is of the form e^{zt} for $z = 1$, and because $z = 1$ is not a root of the characteristic polynomial, a particular solution can be found quickly by either the method of undetermined coefficients (as in the book) or the method of determined coefficients (as in class).

The method of *undetermined coefficients* seeks a particular solution of the form $y_P(t) = Ae^t$. Because $y_P''(t) = Ae^t$, one sees that

$$Ly_P = y_P'' + y_P = 2Ae^t = 4e^t\,,$$

which implies $A = 2$. Hence, $y_P(t) = 2e^t$.

The method of *determined coefficients* evaluates the KEY identity, $Le^{zt} = P(z)e^{zt}$, at $z = 1$ to obtain $Le^t = 2e^t$. Multiplying this by 2 gives $L(2e^t) = 4e^t$, which shows that $y_P(t) = 2e^t$.

By either method you find $y_P(t) = 2e^t$, and the general solution of the problem is therefore

$$y(t) = c_1 \cos(t) + c_2 \sin(t) + 2e^t\,.$$

Because

$$y'(t) = -c_1 \sin(t) + c_2 \cos(t) + 2e^t\,,$$

when the initial conditions are imposed, one finds that

$$y(0) = c_1 + 2 = 0\,, \qquad y'(0) = c_2 + 2 = 0\,.$$

These are solved to find that $c_1 = -2$ and $c_2 = -2$. The solution of the initial-value problem is therefore

$$y(t) = -2\cos(t) + -2\sin(t) + 2e^t\,.$$

(3) Find a general solution for each of the following equations.

(a) $y'' + 4y' + 5y = 3\cos(2t)\,.$

Solution: This is a constant coefficient, inhomogeneous linear problem. The characteristic polynomial of its homogeneous part is

$$P(z) = z^2 + 4z + 5 = (z+2)^2 + 1\,.$$

It has the conjugate pair of roots $-2 \pm i$, which yields the general homogeneous solution

$$y_H(t) = c_1 e^{-2t} \cos(t) + c_2 e^{-2t} \sin(t)\,.$$

Because the forcing is of the form e^{zt} for $z = i2$, and because $z = i2$ is not a root of the characteristic polynomial, a particular solution can be found quickly by either the method of undetermined coefficients (as in the book) or the method of determined coefficients (as in class).

The method of *undetermined coefficients* seeks a particular solution of the form $y_P(t) = A\cos(2t) + B\sin(2t)$. Because

$$y_P'(t) = -2A\sin(2t) + 2B\cos(2t)\,,$$
$$y_P''(t) = -4A\cos(2t) - 4B\sin(2t)\,,$$

one sees that

$$\begin{aligned} Ly_P &= y_P'' + 4y_P' + 5y_P \\ &= (A+8B)\cos(2t) + (B-8A)\sin(2t) = 3\cos(2t)\,. \end{aligned}$$

This leads to the algebraic linear system

$$A + 8B = 3\,, \qquad B - 8A = 0\,.$$

This can be solved to find that $A = 3/65$ and $B = 24/65$. Hence,

$$y_P(t) = \frac{3}{65}\cos(2t) + \frac{24}{65}\sin(2t)\,.$$

The method of *determined coefficients* evaluates the KEY identity, $Le^{zt} = P(z)e^{zt}$, at $z = i2$ to obtain $Le^{i2t} = (1+i8)e^{i2t}$. Multiplying this by $3/(1+i8)$ shows that

$$L\left(\frac{3}{1+i8}e^{i2t}\right) = 3e^{i2t}\,.$$

Because $3e^{i2t} = 3\cos(2t) + i3\sin(2t)$, the real part of the left-hand side above will be Ly_P. Because

$$\begin{aligned} \frac{3}{1+i8}e^{i2t} &= \frac{3(1-i8)}{65}\big(\cos(2t) + i\sin(2t)\big) \\ &= \left(\frac{3}{65}\cos(2t) + \frac{24}{65}\sin(2t)\right) \\ &\quad + i\left(-\frac{24}{65}\cos(2t) + \frac{3}{65}\sin(2t)\right), \end{aligned}$$

this real part shows that

$$y_P(t) = \mathrm{Re}\left(\frac{3}{1+i8}e^{i2t}\right) = \frac{3}{65}\cos(2t) + \frac{24}{65}\sin(2t)\,.$$

By either method you find the same y_P, and the general solution of the problem is therefore

$$y = c_1e^{-2t}\cos(t) + c_2e^{-2t}\sin(t) + \frac{3}{65}\cos(2t) + \frac{24}{65}\sin(2t)\,.$$

(b) $y'' - y = e^t\,.$

Solution: This is a constant coefficient, inhomogeneous linear problem. The characteristic polynomial of its homogeneous part is

$$P(z) = z^2 - 1\,.$$

It the simple real roots -1 and 1, which yields the general homogeneous solution

$$y_H(t) = c_1e^{-t} + c_2e^t\,.$$

Because the forcing is of the form e^{zt} for $z = 1$, and because $z = 1$ is a root of the characteristic polynomial, a particular solution can be found quickly by either the method of undetermined coefficients (as in the book) or the method of determined coefficients (as in class).

The method of *undetermined coefficients* seeks a particular solution of the form $y_P(t) = Ate^t$. Because

$$y_P'(t) = A(e^t + te^t)\,, \qquad y_P''(t) = A(2e^t + te^t)\,,$$

one sees that

$$Ly_P = y_P'' - y_P = A2e^t = e^t\,,$$

which implies $A = 1/2$. Hence, $y_P(t) = \frac{1}{2}te^t$.

The method of *determined coefficients* evaluates the derivative of the KEY identity, $L(te^{zt}) = P'(z)e^{zt} + P(z)te^{zt}$, at $z = 1$ to obtain $L(te^t) = 2e^t$. Dividing this by 2 gives $L(\frac{1}{2}te^t) = e^t$, which shows that $y_P(t) = \frac{1}{2}te^t$.

By either method you find the same y_P, and the general solution of the problem is therefore

$$y(t) = c_1e^{-t} + c_2e^t + \frac{1}{2}te^t\,.$$

(4) Given that x and x^2 are linearly independent solutions of the homogeneous equation

$$x^2y'' - 2xy' + 2y = xe^x\,, \qquad x > 0\,,$$

find a general solution of the equation

$$x^2y'' - 2xy' + 2y = xe^x\,, \qquad x > 0\,.$$

You may express the solution in terms of definite integrals.

Solution: The general solution of this inhomogeneous equation will have the form $y = y_H + y_P$ where y_H is the general solution of the corresponding homogeneous equation and y_P is any particular solution of the inhomogeneous equation. Because you are given that x and x^2 are linearly independent solutions of the corresponding homogeneous equation, you know that

$$y_H = c_1x + c_2x^2\,.$$

The methods of undetermined or determined coefficients cannot be used to find a particular solution, so we will use the method of variation of constants. We first put the equation into its normal form

$$y'' - \frac{2}{x}\,y' + \frac{2}{x^2}\,y = \frac{1}{x}\,e^x\,,$$

and then seek y_P of the form

$$y_P = xu_1(x) + x^2u_2(x)\,.$$

One chooses u_1' and u_2' so that they satisfy

$$xu_1' + x^2u_2' = 0\,, \qquad u_1' + 2xu_2' = \frac{1}{x}\,e^x\,.$$

This linear system is solved to find that

$$u_1' = -\frac{1}{x}\,e^x\,, \qquad u_2' = \frac{1}{x^2}\,e^x\,.$$

Upon integrating these, your answer can be expressed either in terms of indefinite integrals as

$$y = -x \int \frac{1}{x}\, e^x \, dx + x^2 \int \frac{1}{x^2}\, e^x \, dx \,,$$

or in terms of definite integrals as

$$y = c_1 x + c_2 x^2 - x \int_1^x \frac{1}{z}\, e^z \, dz + x^2 \int_1^x \frac{1}{z^2}\, e^z \, dz \,.$$

(5) What answer will be produced by the following MATLAB commands?

```
>> ode1 = 'D2y + 2*Dy + 5*y = 16*exp(t)';
>> dsolve(ode1, 't')
ans =
```

Solution: These commands ask MATLAB to give the general solution of the equation

$$y'' + 2y' + 5y = 16e^t \,.$$

MATLAB produces the answer

```
2*exp(t)+C1*exp(-t)*sin(2*t)+C2*exp(-t)*cos(2*t)
```

This can be seen as follows. This is a constant coefficient, inhomogeneous linear equation. The characteristic polynomial of its homogeneous part is

$$P(z) = z^2 + 2z + 5 = (z+1)^2 + 4 \,.$$

Its roots are $z = -1 \pm i2$, whereby the general solution of the homogeneous part is

$$y_H(t) = c_1 e^{-t} \cos(2t) + c_2 e^{-t} \sin(2t) \,.$$

Because the forcing is of the form e^{zt} for $z = 1$, and because $z = 1$ is not a root of the characteristic polynomial, a particular solution can be found quickly by either the method of undetermined coefficients (as in the book) or the method of determined coefficients (as in class).

The method of *undetermined coefficients* seeks a particular solution of the form $y_P(t) = Ae^t$. Because $y_P'(t) = y_P''(t) = Ae^t$, one sees that

$$Ly_P = y_P'' + 2y_P' + 5y_P = 8Ae^t = 16e^t \,,$$

which implies $A = 2$. Hence, $y_P(t) = 2e^t$.

The method of *determined coefficients* evaluates the KEY identity, $Le^{zt} = P(z)e^{zt}$, at $z = 1$ to obtain $Le^t = 8e^t$. Multiplying this by 2 gives $L(2e^t) = 16e^t$, which shows that $y_P(t) = 2e^t$.

By either method you find $y_P(t) = 2e^t$, and the general solution of the problem is therefore

$$y(t) = c_1 e^{-t} \cos(2t) + c_2 e^{-t} \sin(2t) + 2e^t \,.$$

Up to notational differences, this is the answer that MATLAB produces.

(6) The vertical displacement of a mass on a spring is given by

$$z(t) = \sqrt{3}\cos(2t) + \sin(2t)\,.$$

Express this in the form $z(t) = A\cos(\omega t - \delta)$, identifying the amplitude and phase of the oscillation.

Solution: The displacement takes the form

$$z(t) = 2\cos\left(2t - \tfrac{\pi}{6}\right),$$

where the amplitude is 2 and the phase is $\frac{\pi}{6}$. There are several approaches to this problem. Here are two.

One approach that requires no memorization other than the usual addition formula for cosine is as follows. Because

$$A\cos(\omega t - \delta) = A\cos(\delta)\cos(\omega t) + A\sin(\delta)\sin(\omega t)\,,$$

this form will be equal to $z(t)$ provided $\omega = 2$ and

$$A\cos(\delta) = \sqrt{3}\,, \qquad A\sin(\delta) = 1\,.$$

Upon solving these equations one finds that the amplitude A is given by

$$A = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2\,,$$

while the phase δ is given by

$$\delta = \sin^{-1}\left(\frac{1}{A}\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\,.$$

Another approach requires you to memorize special formulas for both the amplitude and phase of functions of the form

$$c_1\cos(\omega t) + c_2\sin(\omega t)\,.$$

The formula for the amplitude is easier one because c_1 and c_2 appear in it symmetrically. It gives

$$A = \sqrt{c_1^2 + c_2^2} = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2\,.$$

The formula for the phase is trickier because c_1 and c_2 do not appear in it symmetrically. It gives

$$\delta = \tan^{-1}\left(\frac{c_2}{c_1}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}\,.$$

The most common mistake made by those who chose this approach was to exchange the roles of c_1 and c_2 in this formula. One way to keep these roles straight is to remember the formula verbally as

$$\text{phase} = \tan^{-1}\left(\frac{\text{coefficient of sine}}{\text{coefficient of cosine}}\right).$$

(7) When a mass of 4 grams is hung vertically from a spring, at rest it stretches the spring 9.8 cm. (Gravitational acceleration is $g = 980$ cm/sec^2.) At $t = 0$ the mass is displaced 3 cm above its equilibrium position and released with no initial velocity. It moves in a medium that imparts a drag force of 2 dynes (1 dyne = 1 gram cm/sec^2) when the speed of the mass is 4 cm/sec. There are no other forces. (As usual, assume the spring force is proportional to displacement and the drag force is proportional to velocity.)

(a) Formulate an initial-value problem that governs the motion of the mass for $t > 0$. (DO NOT solve the initial-value problem, just write it down!)

Solution: Let y be the displacement of the mass from the equilibrium position in centimeters, with upward displacements being positive. The governing initial-value problem then has the form

$$my'' + \gamma y' + ky = 0\,, \qquad y(0) = 3\,, \quad y'(0) = 0\,,$$

where m is the mass, γ is the drag coefficient, and k is the spring constant. The problem says that $m = 4$ grams. The spring constant is obtained by balancing the weight of the mass ($mg = 4 \cdot 980$ dynes) with the force applied by the spring when it is stretched 9.8 cm. This gives $k9.8 = 4 \cdot 980$, or

$$k = \frac{4 \cdot 980}{9.8} = 400 \quad \text{g/sec}^2\,.$$

The drag coefficient is obtained by balancing the force of 2 dynes with the drag force imparted by the medium when the speed of the mass is 4 cm/sec. This gives $\gamma 4 = 2$, or

$$\gamma = \frac{2}{4} = \frac{1}{2} \quad \text{g/sec}\,.$$

The governing initial-value problem is therefore

$$4y'' + \frac{1}{2}y' + 400y = 0\,, \qquad y(0) = 3\,, \quad y'(0) = 0\,.$$

If you had chosen downward displacements to be positive then the governing initial-value problem would be identical except for the first initial condition, which would then be $y(0) = -3$.

(b) What is the natural frequency of the spring?

Solution: The natural frequency of the spring is given by

$$\omega_o = \sqrt{\frac{k}{m}} = \sqrt{\frac{4 \cdot 980}{9.8 \cdot 4}} = \sqrt{100} = 10 \quad 1/\text{sec}\,.$$

(c) Is the system over damped, critically damped, or under damped? Why?

Solution: The characteristic polynomial is

$$P(z) = z^2 + \frac{1}{8}z + 100 = \left(z + \frac{1}{16}\right)^2 + 100 - \frac{1}{16^2}\,,$$

which has complex roots. The system is therefore under damped.

(8) Compute the Laplace transform of $f(t) = te^{3t}$ from its definition.

Solution: Let $F(s) = \mathcal{L}\{f\}$. By the definition of the Laplace transform

$$F(s) = \lim_{b\to\infty} \int_0^b e^{-st} te^{3t}\, dt = \lim_{b\to\infty} \int_0^b te^{(3-s)t}\, dt\,.$$

An integration by parts shows that

$$\begin{aligned}
\int_0^b t\, e^{(3-s)t}\, dt &= t\, \frac{e^{(3-s)t}}{3-s}\bigg|_0^b - \int_0^b \frac{e^{(3-s)t}}{3-s}\, dt \\
&= \left(t\, \frac{e^{(3-s)t}}{3-s} - \frac{e^{(3-s)t}}{(3-s)^2} \right)\bigg|_0^b \\
&= \left(b\, \frac{e^{(3-s)b}}{3-s} - \frac{e^{(3-s)b}}{(3-s)^2} \right) + \frac{1}{(3-s)^2}\,.
\end{aligned}$$

Hence, provided $s > 3$ one has that

$$\begin{aligned}
F(s) &= \lim_{b\to\infty} \left[\left(b\, \frac{e^{(3-s)b}}{3-s} - \frac{e^{(3-s)b}}{(3-s)^2} \right) + \frac{1}{(3-s)^2} \right] \\
&= \frac{1}{(3-s)^2} + \lim_{b\to\infty} \left(b\, \frac{e^{-(s-3)b}}{3-s} - \frac{e^{-(s-3)b}}{(3-s)^2} \right) \\
&= \frac{1}{(3-s)^2}\,.
\end{aligned}$$

(9) Find the Laplace transform $Y(s)$ of the solution $y(t)$ of the initial-value problem

$$y'' + 4y' + 13y = f(t)\,, \qquad y(0) = 4\,, \quad y'(0) = 1\,,$$

where

$$f(t) = \begin{cases} \cos(t) & \text{for } 0 \le t < 2\pi\,, \\ 0 & \text{for } t \ge 2\pi\,. \end{cases}$$

You may refer to the table on the last page. (DO NOT take the inverse Laplace transform to find $y(t)$, just solve for $Y(s)$.)

Solution: The Laplace transform of the initial-value problem is

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 13\mathcal{L}\{y\} = \mathcal{L}\{f\}\,,$$

where

$$\begin{aligned} \mathcal{L}\{y\} &= Y(s)\,, \\ \mathcal{L}\{y'\} &= sY(s) - y(0) = sY(s) - 4\,, \\ \mathcal{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0) = s^2Y(s) - s4 - 1\,. \end{aligned}$$

To compute $\mathcal{L}\{f\}$, first rewrite f as

$$f(t) = \big(1 - u(t-2\pi)\big)\cos(t) = \cos(t) - u(t-2\pi)\cos(t-2\pi)\,.$$

Referring to the table on the last page, Item 2 with $b = 1$ and Item 5 with $c = 2\pi$ then show that

$$\begin{aligned} \mathcal{L}\{f\} &= \mathcal{L}\{\cos(t)\} - \mathcal{L}\{u(t-2\pi)\cos(t-2\pi)\} \\ &= \frac{s}{s^2+1} - e^{-2\pi s}\frac{s}{s^2+1} \\ &= \big(1 - e^{-2\pi s}\big)\,\frac{s}{s^2+1}\,. \end{aligned}$$

The Laplace transform of the initial-value problem then becomes

$$\big(s^2Y(s) - 4s - 1\big) + 4\big(sY(s) - 4\big) + 13Y(s) = \big(1 - e^{-2\pi s}\big)\,\frac{s}{s^2+1}\,,$$

which becomes

$$(s^2 + 4s + 13)Y(s) - (4s + 1 + 16) = \big(1 - e^{-2\pi s}\big)\,\frac{s}{s^2+1}\,.$$

Hence, $Y(s)$ is given by

$$Y(s) = \frac{1}{s^2 + 4s + 13}\left(4s + 17 + \big(1 - e^{-2\pi s}\big)\,\frac{s}{s^2+1}\right).$$

(10) Find the inverse Laplace tramsform of the following functions. You may refer to the table below (on the last page).

(a) $F(s) = \dfrac{2}{(s+5)^3}$,

Solution: Referring to the table on the last page, Item 1 with $n = 2$ gives $\mathcal{L}\{t^2\} = 2/s^3$. Item 4 with $a = -5$ and $f(t) = t^2$ then gives

$$\mathcal{L}\{e^{-5t}t^2\} = \frac{2}{(s+5)^3}\,.$$

One therefore finds that

$$\mathcal{L}^{-1}\left\{\frac{2}{(s+5)^3}\right\} = e^{-5t}t^2\,.$$

(b) $F(s) = \dfrac{3s}{s^2 - s - 6}$,

Solution: The denominator factors as $(s-3)(s+2)$ so the partial fraction decomposition is

$$F(s) = \frac{3s}{s^2 - s - 6} = \frac{3s}{(s-3)(s+2)} = \frac{\frac{9}{5}}{s-3} + \frac{\frac{6}{5}}{s+2}\,.$$

Referring to the table on the last page, Item 1 with $n = 0$ gives $\mathcal{L}\{1\} = 1/s$. Item 4 with $a = 3$, $a = -2$ and $f(t) = 1$ then gives

$$\mathcal{L}\{e^{3t}\} = \frac{1}{s-3}\,, \qquad \mathcal{L}\{e^{-2t}\} = \frac{1}{s+2}\,.$$

One therefore finds that

$$\mathcal{L}^{-1}\left\{\frac{3s}{s^2 - s - 6}\right\} = \tfrac{9}{5}e^{3t} + \tfrac{6}{5}e^{-2t}\,.$$

(c) $F(s) = \dfrac{(s-2)e^{-3s}}{s^2 - 4s + 5}$.

Solution: Completion of the square in the denominator gives $(s-2)^2 + 1$. Referring to the table on the last page, Item 2 with $b = 1$ gives

$$\mathcal{L}\{\cos(t)\} = \frac{s}{s^2+1}\,.$$

Item 4 with $a = 2$ and $f(t) = \cos(t)$ then gives

$$\mathcal{L}\{e^{2t}\cos(t)\} = \frac{s-2}{(s-2)^2+1}\,.$$

Item 5 with $c = 3$ and $f(t) = e^{2t}\cos(t)$ then gives

$$\mathcal{L}\{u(t-3)e^{2(t-3)}\cos(t-3)\} = e^{-3s}\frac{s-2}{(s-2)^2+1}\,.$$

One therefore finds that

$$\mathcal{L}^{-1}\left\{\frac{(s-2)e^{-3s}}{s^2 - 4s + 5}\right\} = u(t-3)e^{2(t-3)}\cos(t-3)\,.$$

A Short Table of Laplace Transforms

1.	$\mathcal{L}\{t^n\} = \dfrac{n!}{s^{n+1}}$	for $s > 0$.
2.	$\mathcal{L}\{\cos(bt)\} = \dfrac{s}{s^2 + b^2}$	for $s > 0$.
3.	$\mathcal{L}\{\sin(bt)\} = \dfrac{b}{s^2 + b^2}$	for $s > 0$.
4.	$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$	where $F(s) = \mathcal{L}\{f(t)\}$.
5.	$\mathcal{L}\{u(t-c) f(t-c)\} = e^{-cs} F(s)$	where $F(s) = \mathcal{L}\{f(t)\}$ and u is the step function.