Quiz 1 Solutions, Math 246, Professor David Levermore
Wednesday, 6 February 2008

(1) [4] Determine the order of the given differential equation; also state whether the equation is linear or nonlinear:

(a) $t^2\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} + t\,\dfrac{\mathrm{d}y}{\mathrm{d}t} + 2y = \sin(t)$ **Solution:** second order, linear

(b) $\dfrac{\mathrm{d}^2z}{\mathrm{d}t^2} + \sin(z+t) = \sin(t)$ **Solution:** second order, nonlinear

(2) [6] Solve the given initial-value problems:

(a) $t\,\dfrac{\mathrm{d}x}{\mathrm{d}t} + 2x = 4t^2\,, \qquad x(1) = 2$

Solution: This problem is linear, so bring it to the normal form

$$\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{2}{t}\,x = 4t\,.$$

The integrating factor is $e^{A(t)}$ where $A'(t) = 2/t$. Setting $A(t) = 2\log(t)$, we find that $e^{A(t)} = t^2$. Hence, the problem has the intergating factor form

$$\frac{\mathrm{d}}{\mathrm{d}t}\big(t^2x\big) = 4t^3\,.$$

Integrating both sides of this equation yields

$$t^2x = t^4 + c\,.$$

Applying the initial conditon gives

$$1^2 2 = 1^4 + c\,,$$

which implies $c = 1$. The solution is therefore

$$x = t^2 + \frac{1}{t^2}\,.$$

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x}{y}\,, \qquad y(0) = 2$

Solution: This problem is separable. Its separated form is

$$y\,\mathrm{d}y = x\,\mathrm{d}x\,.$$

Integrating both sides yields

$$\tfrac{1}{2}y^2 = \tfrac{1}{2}x^2 + c\,.$$

Applying the initial conditon gives

$$\tfrac{1}{2}2^2 = \tfrac{1}{2}0^2 + c\,,$$

which implies $c = 2$. The solution is therefore

$$y = \sqrt{x^2 + 4}\,,$$

where the positive square root is taken to satisfy the initial condition.