

Quiz 3 Solutions, Math 246, Professor David Levermore
Wednesday, 20 February 2008

- (1) [2] Suppose you are using the Runge-Kutta method to numerically approximate the solution of an initial-value problem over the time interval $[0, 5]$. By what factor would you expect the error to decrease when you increase the number of time steps taken from 100 to 300.

Solution: When you increase the number of time steps by a factor of 3, the time step h is reduced by a factor of 3. Because the Runge-Kutta method is fourth order, the error will therefore decrease by a factor of $3^4 = 81$.

- (2) [4] Consider the following MATLAB function M-file.

```
function [t,y] = solveit(to, yo, tf, n)

h = (tf - to)/n;
t = zeros(n + 1, 1);
y = zeros(n + 1, 1);
t(1) = to;
y(1) = yo;
for i = 1:n
z = t(i)^4 + y(i)^2;
t(i + 1) = t(i) + h;
y(i + 1) = y(i) + (h/2)*(z + t(i + 1)^4 + (y(i) + h*z)^2);
end
```

- (a) What is the initial-value problem being approximated numerically?
(b) What is the numerical method being used?

Solution: The initial-value problem being approximated is

$$\frac{dy}{dt} = t^4 + y^2, \quad y(t_0) = y_0.$$

The Heun-Trapezoidal (improved Euler) method is being used.

- (3) [4] Consider the initial-value problem

$$\frac{dz}{dt} = z - z^2, \quad z(0) = 2.$$

Use the Euler method with $h = .1$ to compute approximations of $z(.1)$ and $z(.2)$. Leave your answer for $z(.2)$ as an arithmetic expression.

Solution: Set $z_0 = z(0) = 2$. The Euler method then yields

$$\begin{aligned} z(.1) &\approx z_1 = z_0 + h(z_0 - z_0^2) = 2 + .1(2 - 2^2) = 2 + .1(-2) = 1.8, \\ z(.2) &\approx z_2 = z_1 + h(z_1 - z_1^2) = 1.8 + .1(1.8 - (1.8)^2). \end{aligned}$$