Sample Problems for Second In-Class Exam
Math 246, Fall 2008, Professor David Levermore

(1) Give the interval of existence for the solution of the initial-value problem

$$\frac{\mathrm{d}^3x}{\mathrm{d}t^3} + \frac{\cos(3t)}{4-t}\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{e^{-2t}}{1+t}, \qquad x(2) = x'(2) = x''(2) = 0.$$

(2) Let L be a linear ordinary differential operator with constant coefficients. Suppose that all the roots of its characteristic polynomial (listed with their multiplicities) are $-2+i3$, $-2-i3$, $i7$, $i7$, $-i7$, $-i7$, 5, 5, 5, -3, 0, 0.
(a) Give the order of L.
(b) Give a general real solution of the homogeneous equation $\mathrm{L}y = 0$.

(3) Let $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}$. Solve each of the following initial-value problems.
(a) $\mathrm{D}^2y + 4\mathrm{D}y + 4y = 0, \qquad y(0) = 1, \quad y'(0) = 0.$
(b) $\mathrm{D}^2y + 9y = 20e^t, \qquad y(0) = 0, \quad y'(0) = 0.$

(4) Let $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}$. Give a general real solution for each of the following equations.
(a) $\mathrm{D}^2y + 4\mathrm{D}y + 5y = 3\cos(2t)$.
(b) $\mathrm{D}^2y - y = t\,e^t$.
(c) $\mathrm{D}^2y - y = \dfrac{1}{1+e^t}$.

(5) Let $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{d}t}$. Consider the equation

$$\mathrm{L}y = \mathrm{D}^2y - 6\mathrm{D}y + 25y = e^{t^2}.$$

(a) Compute the Green function $g(t)$ associated with L.
(b) Use the Green function to express a particular solution $Y_P(t)$ in terms of definite integrals.

(6) The functions x and x^2 are solutions of the homogeneous equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 0 \qquad \text{over } x > 0.$$

(You do not have to check that this is true!)
(a) Compute their Wronskian.
(b) Give a general solution of the equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = x^3e^x \qquad \text{over } x > 0.$$

You may express the solution in terms of definite integrals.

(7) What answer will be produced by the following MATLAB commands?

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>> ode1 = 'D2y + 2*Dy + 5*y = 16*exp(t)';
>> dsolve(ode1, 't')
ans =
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(8) The vertical displacement of a mass on a spring is given by

$$z(t) = \sqrt{3}\cos(2t) + \sin(2t)\,.$$

Express this in the form $z(t) = A\cos(\omega t - \delta)$, identifying the amplitude and phase of the oscillation.

(9) When a mass of 4 grams is hung vertically from a spring, at rest it stretches the spring 9.8 cm. (Gravitational acceleration is $g = 980$ cm/sec^2.) At $t = 0$ the mass is displaced 3 cm above its equilibrium position and is released with no initial velocity. It moves in a medium that imparts a drag force of 2 dynes (1 dyne = 1 gram cm/sec^2) when the speed of the mass is 4 cm/sec. There are no other forces. (Assume that the spring force is proportional to displacement and that the drag force is proportional to velocity.)

(a) Formulate an initial-value problem that governs the motion of the mass for $t > 0$. (DO NOT solve this initial-value problem, just write it down!)

(b) What is the natural frequency of the spring?

(c) Show that the system is under damped and find its quasifrequency.