Quiz 3 Solutions, Math 246, Professor David Levermore
Tuesday, 23 September 2008

(1) [2] Suppose you are using the Runge-Kutta method to numerically approximate the solution of an initial-value problem over the time interval $[0, 5]$. By what factor would you expect the global error to decrease when you increase the number of time steps taken from 200 to 600.

Solution: When you increase the number of time steps by a factor of 3, the time step h is reduced by a factor of 3. Because the Runge-Kutta method is fourth order, the error will therefore decrease by a factor of $3^4 = 81$.

(2) [4] Consider the following MATLAB function M-file.

```
function [t,y] = solveit(ti, yi, tf, n)

h = (tf - ti)/n;
t = zeros(n + 1, 1);
y = zeros(n + 1, 1);
t(1) = ti;
y(1) = yi;
for j = 1:n
z = t(j)^2 + y(j)^4;
t(j + 1) = t(j) + h;
y(j + 1) = y(j) + (h/2)*(z + t(j + 1)^2 + (y(j) + h*z)^4);
end
```

(a) What is the initial-value problem being approximated numerically?
(b) What is the numerical method being used?

Solution: The initial-value problem being approximated is

$$\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 + y^4 \,, \qquad y(t_i) = y_i \,.$$

The Heun-Trapezoidal (improved Euler) method is being used.

(3) [4] Consider the initial-value problem

$$\frac{\mathrm{d}w}{\mathrm{d}t} = w^2 - 2w \,, \qquad w(0) = 3 \,.$$

Use the forward Euler method with $h = .1$ to compute approximations of $w(.1)$ and $w(.2)$. Leave your answer for $z(.2)$ as an arithmetic expression.

Solution: Set $w_0 = w(0) = 3$. The Euler method then yields

$$w(.1) \approx w_1 = w_0 + h(w_0^2 - 2w_0) = 3 + .1(3^2 - 2 \cdot 3) = 3 + .1(9 - 6) = 3.3 \,,$$
$$w(.2) \approx w_2 = w_1 + h(w_1^2 - 2w_1) = 3.3 + .1\big((3.3)^2 - 2 \cdot 3.3\big) \,.$$