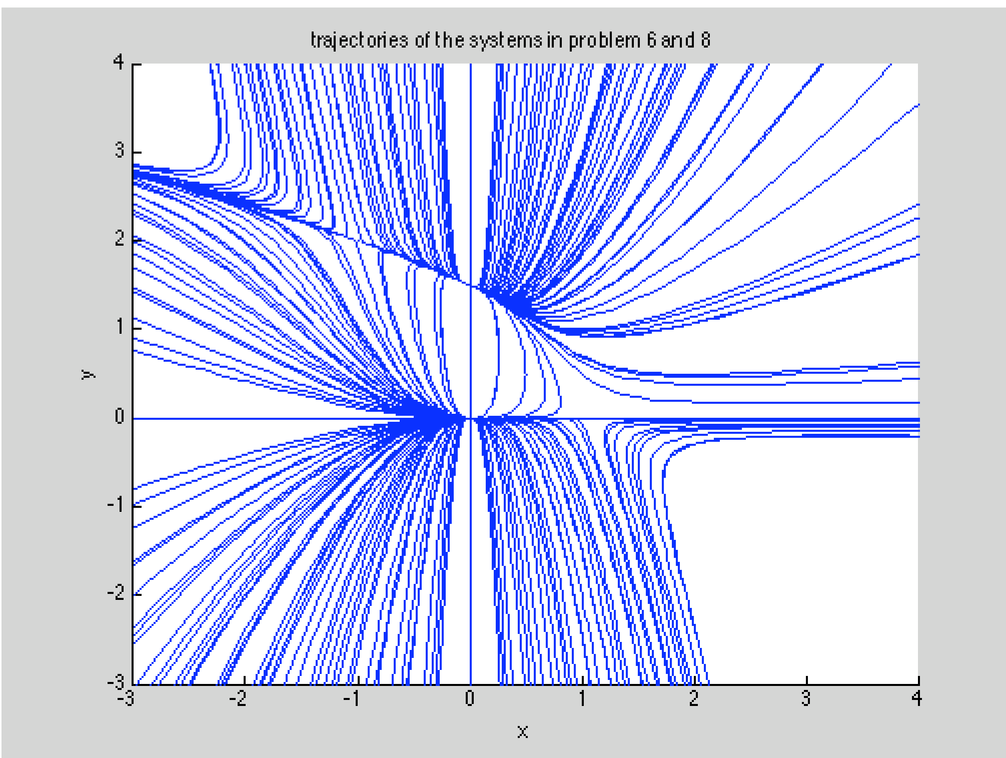
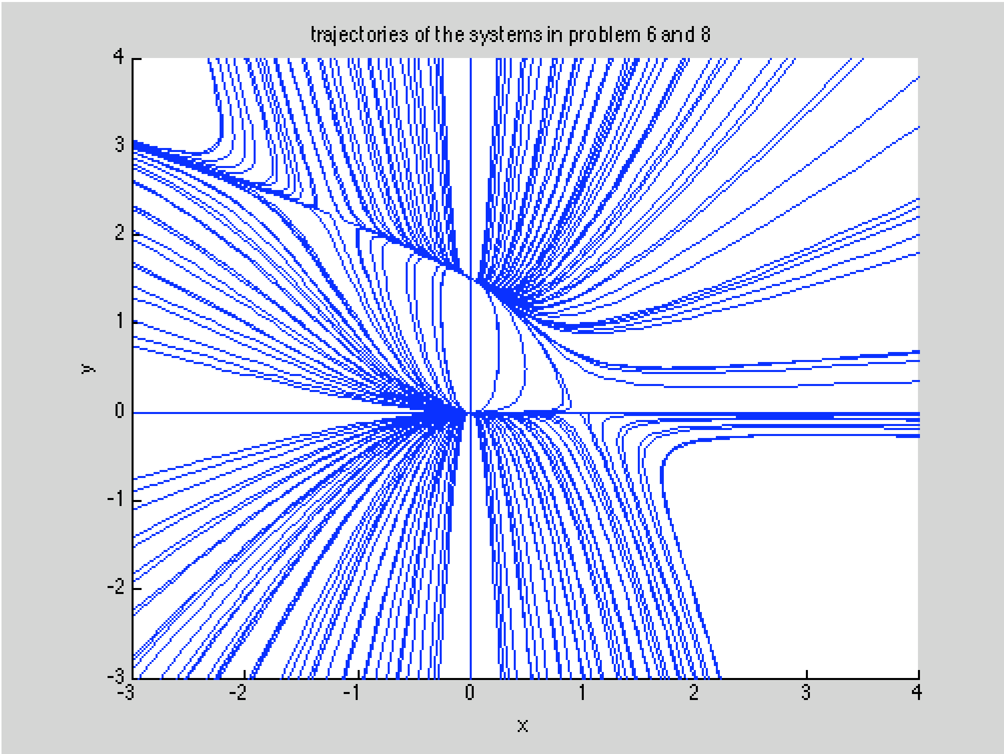
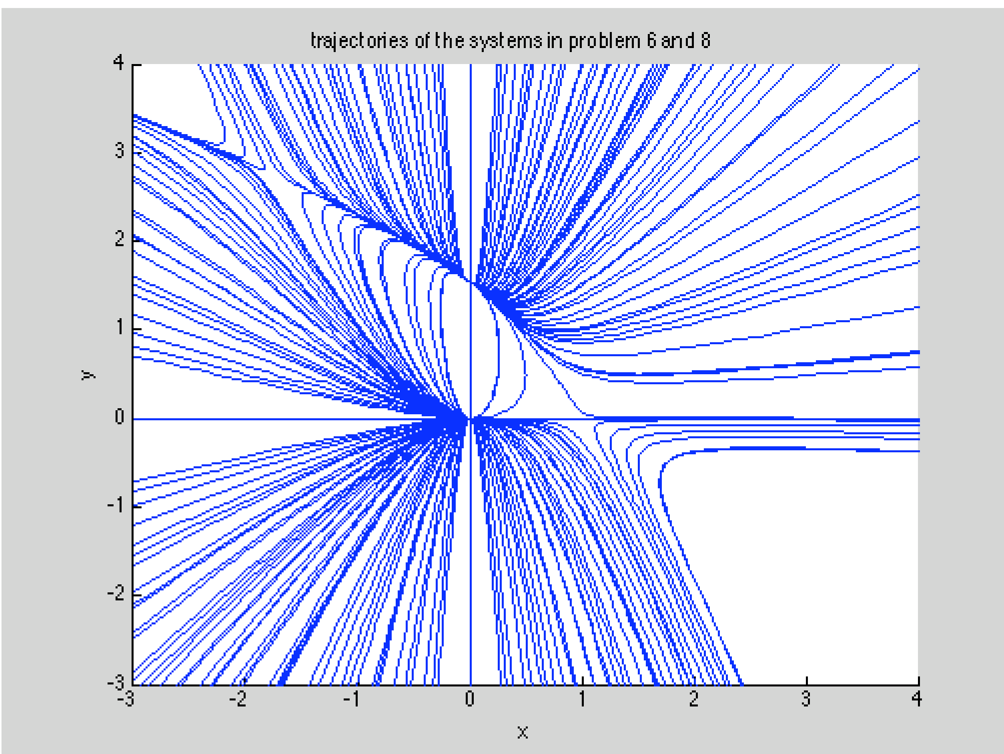
Contents

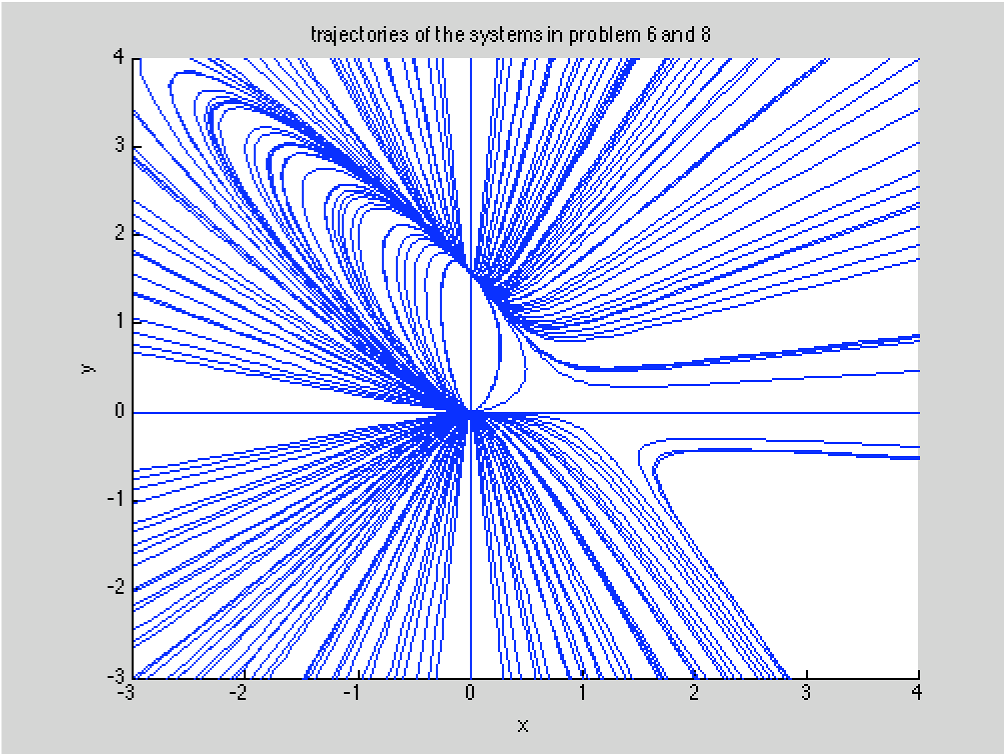

```
% Abdulmalik Almeheini
% MATH246 extracredit HW
% problems 6 and 8 from sec 9.3
```

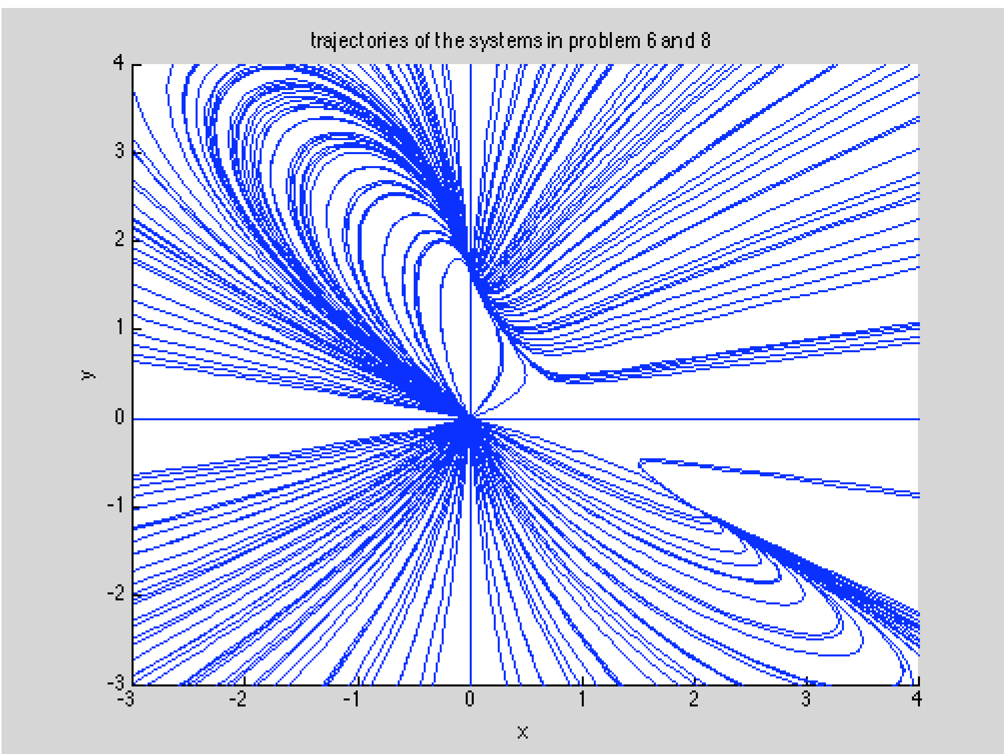
```
clear
clc
warning off all
for alph = 0:0.2:1
f = @(t, x) (1-alph)*[x(1)*(1 - x(1) - x(2));
            x(2)*(3 -  x(1) - 2*x(2))] + alph*[x(1)*(1 - x(1) - x(2));
            x(2)*(1/2 - (3/4)*x(1) - (1/4)*x(2))];
figure; hold on
for a = -2.25:0.25:1.75
    for b = -2.5:0.5:4
        [t, xa] = ode45(f, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis([-3 4 -3 4])
xlabel x
ylabel y
title 'trajectories of the systems in problem 6 and 8'
end
```

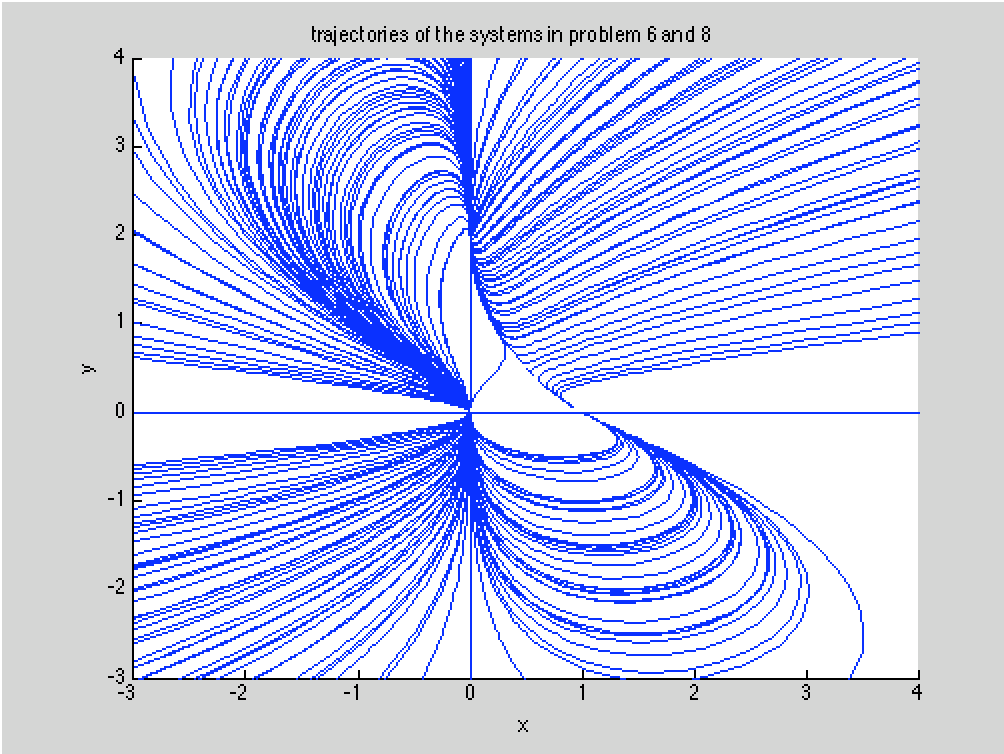



trajectories of the systems in problem 6 and 8
y
x
4
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
4


trajectories of the systems in problem 6 and 8
y
x
4
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3
4


trajectories of the systems in problem 6 and 8
y
x


trajectories of the systems in problem 6 and 8
y
x




```
% From the graphs above we can see how the systems in problem 6 change to
% become the systems in problem 8 as alpha changes from 0 to 1. The reason
% behind using alpha is to see how the systems behave as they change
% from a state to another( which the two states here are problems 6 and 8.)
```

First (stationary/critical point locations) :

```
% The first graph shows the trajectories of the systems  when alpha = 0 which is
% exactly the trajectories of the systems in problem 6.
% As we can see from the first graph that the system when alpha =0
% has 4 critical points at (0,0),(1,0),(0,3/2),and(-1,2). And when we jump
% to see the systems when the alpha = 1 we can see that it still have 4
% critical points , however, two of them are different and two are the same
% (0,0) and (1,0).
% And to see how the systems behaved while changing from problem 6 to
% problem 8 we change the value of alpha from 0 to 1 with increment of 0.2.
% this changing in the value give us four more graphs that tell us the story
% behind these systems.
% if we look at the systems trajectories when alpha is 0.2, 0.4 and 0.6, we
% can see the all of them share two of the critical points ( 0,0) and(1,0).
% However, they all have different values for the other two critical
% points. one thing we can notice in these different critical points is that
% as alpha goes from 0 to 0.6 they are changing every time to the same
% directions, for example, when alpha = 0 the systems started with
% (0,0),(1,0),(0,3/2)and(-1,2), and as  alpha increased to 0.6 two of them
% changed, the critical point (0,3/2) changed to (0,50/33) then
% (0,20/13) then at alpha = 0.6 to (0,30/19) and the critical point
% (-1,2) changed to ( -17/14,31/14) then (-7/4,11/4) then (-11/2,13/2), we
% can see that the first one was moving along the positive y axis and the
% other one was moving along the positive y axis and along the
% negative x axis like moving toward the corner.
% However, when alpha changes to 0.8 and then to 1 something strange
% happens to the systems. This strange thing occurs to one of the critical
% points. we know that two of the critical points were moving and each one was moving at a specific direction,
% however, when alpha changes to 0.8 the critical point (-11/2, 13/2) moves all the way from
% the second quadrant to the fourth quadrant to become ( 2,-1)
% then when alpha changes to 1 it jumps to the first quadrant to become (1/2,1/2)
% thats how these systems went from the first state to the second. And to illustrate these changes
% I solved for the critical points for the systems with each different value of alpha.
```

These are the critical/stationary points and corrisponding solution for the systems as alpha goes from 0 to 1:

```
clear
clc
for alph = 0:0.2:1
syms x y
```

```
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc])

A = jacobian([S1 S2], [x y]);
evals = eig(A);
end
```

```
Critical points:
[  0,   0]
[  1,   0]
[  0, 3/2]
[ -1,   2]

Critical points:
[      0,     0]
[      1,     0]
[      0, 50/33]
[ -17/14, 31/14]

Critical points:
[    0,     0]
[    1,     0]
[    0, 20/13]
[ -7/4,  11/4]

Critical points:
[     0,     0]
[     1,     0]
[     0, 30/19]
[ -11/2,  13/2]

Critical points:
[ 0,   0]
[ 1,   0]
[ 0, 5/3]
[ 2,  -1]

Critical points:
[   0,   0]
[   0,   2]
[   1,   0]
[ 1/2, 1/2]
```

Second: now we solve for the eigen values for the systems to show how the stability of these points change as alpha change

```
clear
clc

% when alpha = 0
syms x y
alph = 0;
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc]);

A = jacobian([S1 S2], [x y]);
evals = eig(A)

disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (1,0);');
disp(double(subs(evals, {x, y}, {1, 0})))
disp('Eigenvalues at (0,3/2);');
disp(double(subs(evals, {x, y}, {0, 3/2})))
disp('Eigenvalues at (-1,2);');
disp(double(subs(evals, {x, y}, {-1, 2})))
```

```
Critical points:
[  0,   0]
[  1,   0]
[  0, 3/2]
[ -1,   2]


evals =

 2 - (5*y)/2 - (x^2 - 2*x*y + 4*x + 9*y^2 - 12*y + 4)^(1/2)/2 - (3*x)/2
 (x^2 - 2*x*y + 4*x + 9*y^2 - 12*y + 4)^(1/2)/2 - (5*y)/2 - (3*x)/2 + 2

Eigenvalues at (0,0);
     1
```

```
     3

Eigenvalues at (1,0);
    -1
     2

Eigenvalues at (0,3/2);
   -3.0000
   -0.5000

Eigenvalues at (-1,2);
   -3.5616
    0.5616
```

```
% From the eigen values we can conclude that at the point ( 0,0) a we have
% a nodel source b/c both eigen values are real and positive values so its unstable, and at the
% points (1,0) and (-1,2) we have saddles, which they are also unstable
% b/c one of the eigen values is negative and the other is positive
% and finally at the point (0,3/2) we have a nodel sink (stable) b/c both eigen values are
% real and negative.
```

```
clear
clc

% when alpha = 0.2
syms x y
alph = 0.2;
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc]);

A = jacobian([S1 S2], [x y]);
evals = eig(A)

disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (1,0);');
disp(double(subs(evals, {x, y}, {1, 0})))
disp('Eigenvalues at (0,50/33);');
disp(double(subs(evals, {x, y}, {0, 50/33})))
disp('Eigenvalues at (-17/14,31/14);');
disp(double(subs(evals, {x, y}, {-17/14, 31/14})))
```

```
Critical points:
[      0,     0]
[      1,     0]
[      0, 50/33]
[ -17/14, 31/14]


evals =

 7/4 - (43*y)/20 - ((441*x^2)/400 - (103*x*y)/100 + (63*x)/20 + (529*y^2)/100 - (69*y)/10 + 9/4)^(1/2)/2 - (59*x)/40
 ((441*x^2)/400 - (103*x*y)/100 + (63*x)/20 + (529*y^2)/100 - (69*y)/10 + 9/4)^(1/2)/2 - (43*y)/20 - (59*x)/40 + 7/4

Eigenvalues at (0,0);
    1.0000
    2.5000

Eigenvalues at (1,0);
   -1.0000
    1.5500

Eigenvalues at (0,50/33);
   -2.5000
   -0.5152

Eigenvalues at (-17/14,31/14);
   -3.0553
    0.6160
```

```
% When we see the aigen values of the systems when alpha = 0.2
% we see that thier stability havent changed, where
% (0,0),(1,0) and (-17/14,31/14) are unstabel and (0,50/33) is stable ,so
% thier locations only have changed .
```

```
clear
clc
```

```
% when alpha = 0.4
syms x y
alph = 0.4;
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc]);

A = jacobian([S1 S2], [x y]);
evals = eig(A)

disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (1,0);');
disp(double(subs(evals, {x, y}, {1, 0})))
disp('Eigenvalues at (0,20/13);');
disp(double(subs(evals, {x, y}, {0, 20/13})))
disp('Eigenvalues at (-7/4,11/4);');
disp(double(subs(evals, {x, y}, {-7/4, 11/4})))
```

```
Critical points:
[    0,     0]
[    1,     0]
[    0, 20/13]
[ -7/4,  11/4]


evals =

 3/2 - (9*y)/5 - ((121*x^2)/100 + (2*x*y)/25 + (11*x)/5 + (64*y^2)/25 - (16*y)/5 + 1)^(1/2)/2 - (29*x)/20
 ((121*x^2)/100 + (2*x*y)/25 + (11*x)/5 + (64*y^2)/25 - (16*y)/5 + 1)^(1/2)/2 - (9*y)/5 - (29*x)/20 + 3/2

Eigenvalues at (0,0);
     1
     2

Eigenvalues at (1,0);
   -1.0000
    1.1000

Eigenvalues at (0,20/13);
   -2.0000
   -0.5385

Eigenvalues at (-7/4,11/4);
   -2.5731
    0.7481
```

```
% also when we see the aigen values of the systems when alpha = 0.4
% we see that thier stability havent changed, where
% (0,0),(1,0) and (-7/4,11/4) are unstabel and (0,20/13) is stable ,so
% thier locations only have changed .
```

```
clear
clc

% when alpha = 0.6
syms x y
alph = 0.6;
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc]);

A = jacobian([S1 S2], [x y]);
evals = eig(A)

disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (1,0);');
disp(double(subs(evals, {x, y}, {1, 0})))
disp('Eigenvalues at (0,30/19);');
disp(double(subs(evals, {x, y}, {0, 30/19})))
disp('Eigenvalues at (-11/2,13/2);');
disp(double(subs(evals, {x, y}, {-11/2,13/2})))
```

```
Critical points:
[     0,     0]
[     1,     0]
[     0, 30/19]
[ -11/2,  13/2]
```

```
evals =

 5/4 - (29*y)/20 - ((529*x^2)/400 + (133*x*y)/100 + (23*x)/20 + (81*y^2)/100 - (9*y)/10 + 1/4)^(1/2)/2 - (57*x)/40
 ((529*x^2)/400 + (133*x*y)/100 + (23*x)/20 + (81*y^2)/100 - (9*y)/10 + 1/4)^(1/2)/2 - (29*y)/20 - (57*x)/40 + 5/4

Eigenvalues at (0,0);
    1.0000
    1.5000

Eigenvalues at (1,0);
   -1.0000
    0.6500

Eigenvalues at (0,30/19);
   -1.5000
   -0.5789

Eigenvalues at (-11/2,13/2);
   -2.2582
    1.5832
```

```
% also when alpha = 0.6 the critical points stability dont change.
```

```
clear
clc

% when alpha = 0.8
syms x y
alph = 0.8;
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc]);

A = jacobian([S1 S2], [x y]);
evals = eig(A)

disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (1,0);');
disp(double(subs(evals, {x, y}, {1, 0})))
disp('Eigenvalues at (0,5/3);');
disp(double(subs(evals, {x, y}, {0, 5/3})))
disp('Eigenvalues at (2,-1);');
disp(double(subs(evals, {x, y}, {2, -1})))
```

```
Critical points:
[ 0,   0]
[ 1,   0]
[ 0, 5/3]
[ 2,  -1]


evals =

 1 - (11*y)/10 - ((36*x^2)/25 + (68*x*y)/25 + y^2/25)^(1/2)/2 - (7*x)/5
 ((36*x^2)/25 + (68*x*y)/25 + y^2/25)^(1/2)/2 - (11*y)/10 - (7*x)/5 + 1

Eigenvalues at (0,0);
     1
     1

Eigenvalues at (1,0);
   -1.0000
    0.2000

Eigenvalues at (0,5/3);
   -1.0000
   -0.6667

Eigenvalues at (2,-1);
   -1.0000
   -0.4000
```

```
% But when we look at the eigen values of the systems when alpha = 0.8 we
% notice that the stability of the critical point that changes it location
% from the second quadrant to the fourth qudrant also changes, which it
% became stable and swiched to nodel sink.  while nothing happens to the
% stability of the other points.
```

```
clear
clc

% when alpha = 1
syms x y
alph = 1;
S1 = x*(1 - x - y);
S2 = (1-alph)*y*(3 - x - 2*y) + alph*y*(1/2 - (3/4)*x - (1/4)*y);
[xc, yc] = solve(S1, S2, x, y);
disp('Critical points:'); disp([xc yc])

A = jacobian([S1 S2], [x y]);
evals = eig(A)

disp('Eigenvalues at (0,0);');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (0,2);');
disp(double(subs(evals, {x, y}, {0, 2})))
disp('Eigenvalues at (1,0);');
disp(double(subs(evals, {x, y}, {1, 0})))
disp('Eigenvalues at (1/2,1/2);');
disp(double(subs(evals, {x, y}, {1/2, 1/2})))
```

```
Critical points:
[   0,   0]
[   0,   2]
[   1,   0]
[ 1/2, 1/2]


evals =

 3/4 - (3*y)/4 - ((25*x^2)/16 + (17*x*y)/4 - (5*x)/4 + y^2/4 - y/2 + 1/4)^(1/2)/2 - (11*x)/8
 ((25*x^2)/16 + (17*x*y)/4 - (5*x)/4 + y^2/4 - y/2 + 1/4)^(1/2)/2 - (3*y)/4 - (11*x)/8 + 3/4

Eigenvalues at (0,0);
    0.5000
    1.0000

Eigenvalues at (0,2);
   -1.0000
   -0.5000

Eigenvalues at (1,0);
   -1.0000
   -0.2500

Eigenvalues at (1/2,1/2);
   -0.7844
    0.1594
```

```
% and when we look at the eigen values of the systems when alpha=1 we also
% see that these changes countinue happening on some of the points. first,
% we know that the point that changed its location from the second to the
% fourth quadrant moves to the first quadrant when alpha = 1, however, what
% we see here that its changes it stability while its moving so it return
% to be unstable again. and another thing is the point (1,0) that havent
% changed at all while alpha changed from 0 to 0.8 finally becomes stable
% when alpha = 1 and i think this changes occuers b/c of the point that
% comes close to it.
```