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%Math 246, Differential Equations - Extra Credit Project
%Tuesday, May 12, 2009

%Assigned Problem:
%d/dt[x; y] = [Hy; -Hx - gamma Hy]
%H(x, y) = 1/2*y^2 + 1/2*x^2 - 1/3*x^3


syms x y
%Setting H, and calculating its partials.
H = 1/2*y^2 + 1/2*x^2 - 1/3*x^3
diff(H,y)
diff(H,x)
%Additionally, I am finding the critical points (assuming gamma=1). Later
%on, at the end of my M-file, I note that the critical points remain the
%same regardless of what gamma is set to.
[x, y] = solve(y, x^2-x-y); [x y]
%Therefore the critical points are (0, 0) and (1, 0).
clear all
%Now setting up the matrix to find the eigenvalues, which in turn will tell
%us if it is a sink, source, saddle, spiral, twist, etc.
syms x y
f = y;
g = x^2-x-y;
A = [diff(f,x) diff(f,y); diff(g,x) diff(g,y)]
%Plugging (0, 0) into A and calculating its eigenvalues.
A = [0 1; -1 -1]
[xi, R] = eig(sym(A))
%When (0, 0) is plugged into matrix A and the eigenvalues are calculated,
%we learn that both eigenvalues are negative and conjugate pairs, thus a
%spiral sink surrounding (0, 0). By the A21 rule, it is clockwise.

%Plugging (1, 0) into A and calculating its eigenvalues.
A = [0 1; 1 -1]
[xi, R] = eig(sym(A))
%Since both eigenvalues are real, with 1 eigenvalue negative and 1
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%positive, we have a saddle surrounding the point (1, 0). By the A21 rule,
%it is counterclockwise (since A21=1>0).

%Below I am plotting a family of solutions/trajectories when gamma=1, using
%ode45.
warning off all
figure, hold on
f = @(t, x) [x(2); x(1)^2-x(1)-x(2)];
for a = -2:2
   for b = -2:2
      [t, xa] = ode45(f, [0 3], [a b]);
      plot(xa(:,1), xa(:,2))
      [t, xa] = ode45(f, [0 -3], [a b]);
      plot(xa(:,1), xa(:,2))
   end
end
axis([-4 8 -15 10])
xlabel 'x'
ylabel 'y'
title 'Solutions when gamma=1 of dx/dt=Hy and dy/dt=-Hx - gamma Hy'
%Conclusion: As previously described above when the eigenvalues were
%calculated, around (0, 0) we see a clockwise spiral sink, and it is stable.
%Slightly to the right, at the point (1, 0), there is a saddle (unstable).

%Now plotting a family of solutions/trajectories when gamma=.7.
warning off all
figure, hold on
f = @(t, x) [x(2); x(1)^2-x(1)-.7*x(2)];
for a = -2:2
   for b = -2:2
      [t, xa] = ode45(f, [0 3], [a b]);
      plot(xa(:,1), xa(:,2))
      [t, xa] = ode45(f, [0 -3], [a b]);
      plot(xa(:,1), xa(:,2))
   end
end
axis([-4 8 -15 10])
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xlabel 'x'
ylabel 'y'
title 'Solutions when gamma=.7 of dx/dt=Hy and dy/dt=-Hx - gamma Hy'
%This illustration shows, when gamma gets slightly less than 1, a
%slight counterclockwise rotation occurs.

%Now plotting a family of solutions/trajectories when gamma=.3.
warning off all
figure, hold on
f = @(t, x) [x(2); x(1)^2-x(1)-.3*x(2)];
for a = -2:2
   for b = -2:2
      [t, xa] = ode45(f, [0 3], [a b]);
      plot(xa(:,1), xa(:,2))
      [t, xa] = ode45(f, [0 -3], [a b]);
      plot(xa(:,1), xa(:,2))
   end
end
axis([-4 8 -15 10])
xlabel 'x'
ylabel 'y'
title 'Solutions when gamma=.3 of dx/dt=Hy and dy/dt=-Hx - gamma Hy'
%This shows the trajectories, maintaining their critical points (and still
%their directions due to gamma being positive), are turning more and more
%counterclockwise as gamma gets closer and closer to 0.

%Now plotting a family of solutions/trajectories when gamma=0.
warning off all
figure, hold on
f = @(t, x) [x(2); x(1)^2-x(1)];
for a = -2:2
   for b = -2:2
      [t, xa] = ode45(f, [0 3], [a b]);
      plot(xa(:,1), xa(:,2))
      [t, xa] = ode45(f, [0 -3], [a b]);
      plot(xa(:,1), xa(:,2))
   end
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end
axis([-4 8 -15 10])
xlabel 'x'
ylabel 'y'
title 'Solutions when gamma=0 of dx/dt=Hy and dy/dt=-Hx - gamma Hy'
%As we have seen in all the previous graphs up to this one, gamma has been
%going from 1 towards 0. As gamma has been approaching 0 (from the right,
%or positive side), the trajectories would rotate more and more
%counterclockwise. Once gamma=0, the trajectories lie symmetric across the
%'y' axis! We now have a part of a clockwise center (only imaginary eigenvalues). As
%depicted in the graph, the top, left and bottom surrounding (0, 0) appear as a center,
%however the critical point (1, 0) still holds the form of a saddle, which
%deflects the full elliptical continuation of the center at (0, 0). Therefore, since it is not
%a fully revolving center, it technically is not a center, and is unstable
%(the saddle ‘pulls’ the right side of the ‘center’ to y=infinity and y=-infinity).

%Now plotting a family of solutions/trajectories when gamma=-1.
warning off all
figure, hold on
f = @(t, x) [x(2); x(1)^2-x(1)+x(2)];
for a = -2:2
   for b = -2:2
      [t, xa] = ode45(f, [0 3], [a b]);
      plot(xa(:,1), xa(:,2))
      [t, xa] = ode45(f, [0 -3], [a b]);
      plot(xa(:,1), xa(:,2))
   end
end
axis([-4 8 -15 10])
xlabel 'x'
ylabel 'y'
title 'Solutions when gamma=-1 of dx/dt=Hy and dy/dt=-Hx - gamma Hy'
%Here we see a rotation of 180 degrees about the y axis when compared to the graph of
%gamma=1.

%Conclusion: It is important to note that gamma can be anything and still not
%affect the critical points. I believe that this is most likely
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%due to the fact that gamma is a constant. Therefore it changes the
%magnitude and direction of the trajectories, but it does not move the critical
%points around; they stay still. Interestingly enough, even when gamma=0 causing a
%'y' variable to be lost (changing from dy/dt=x^2-x-y to dy/dt=x^2-x), this
%still does not change the critical points. What is fascinating is
%how even when gamma is set to -1, the whole image ‘appears’ to be rotated 180 degrees
around the y axis - and the critical points remain the same. Although everything looks the
%same, the direction is now spiraling outward, indicative of a source. I know the critical points
%remain the same regardless of gamma because when I use the 'solve' command with
%gamma=-1, 1, or 0 it still gives me the same critical points. Additionally, it could not be simply
%flipped across the y axis neither, because it would it would still remain a sink and then yield a
%counterclockwise spiral. The A21 rule tells us the spiral remains clockwise, and the
%eigenvalues further the logic of affirming the spiral is now a source.

%Although the critical points will never change, the eigenvalues will,
%meaning the trajectories surrounding the critical points have the potential to change direction
%(depending on whether gamma is positive or negative). Only when gamma is positive, the
%critical point (0, 0) will always be a stable sink. However, when gamma is negative there
%exists a source instead. When gamma is 0 there lies a portion of a ‘center’ surrounding (0,0)
%except on the side of the other critical point (1, 0), which is where the saddle deflects part of
%the ‘center’ and shoots the top part of it towards positive infinity and the bottom part towards
%negative infinity.

%For example, when gamma=-1, it yields a positive conjugate pair of
%eigenvalues for the critical point (0, 0), indicating
%a clockwise unstable spiral source instead! (This makes sense; it is impossible
%to have the same clockwise spiral sink reflected across the y-axis; it logically
%cannot be drawn.) When gamma=-1 for the critical point (1, 0), we still get a saddle
%(both real, 1 positive and 1 negative eigenvalue), however the signs are
%switched on the eigenvalues corresponding to similar eigenvectors. In other words,
%the eigenvalue attracting when gamma=1 is now repelling when gamma=-1, and
%vise versa for the switching of the other eigenvalue as well. This is consistant with the
%source switching to a sink. The code to find the eigenvalues for both critical points
%when gamma=-1 is listed below.

```
clear all
syms x y
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f = y;
%When gamma=-1, this makes 'y' positive below, as opposed to 'y' being
%negative when gamma was positive.
g = x^2-x+y;
A = [diff(f,x) diff(f,y); diff(g,x) diff(g,y)]
%Plugging (0, 0) into A and calculating its eigenvalues.
A = [0 1; -1 1]
[xi, R] = eig(sym(A))
%Now plugging the critical point (1, 0) into A and calculating its
%eigenvalues.
A = [0 1; 1 1]
[xi, R] = eig(sym(A))
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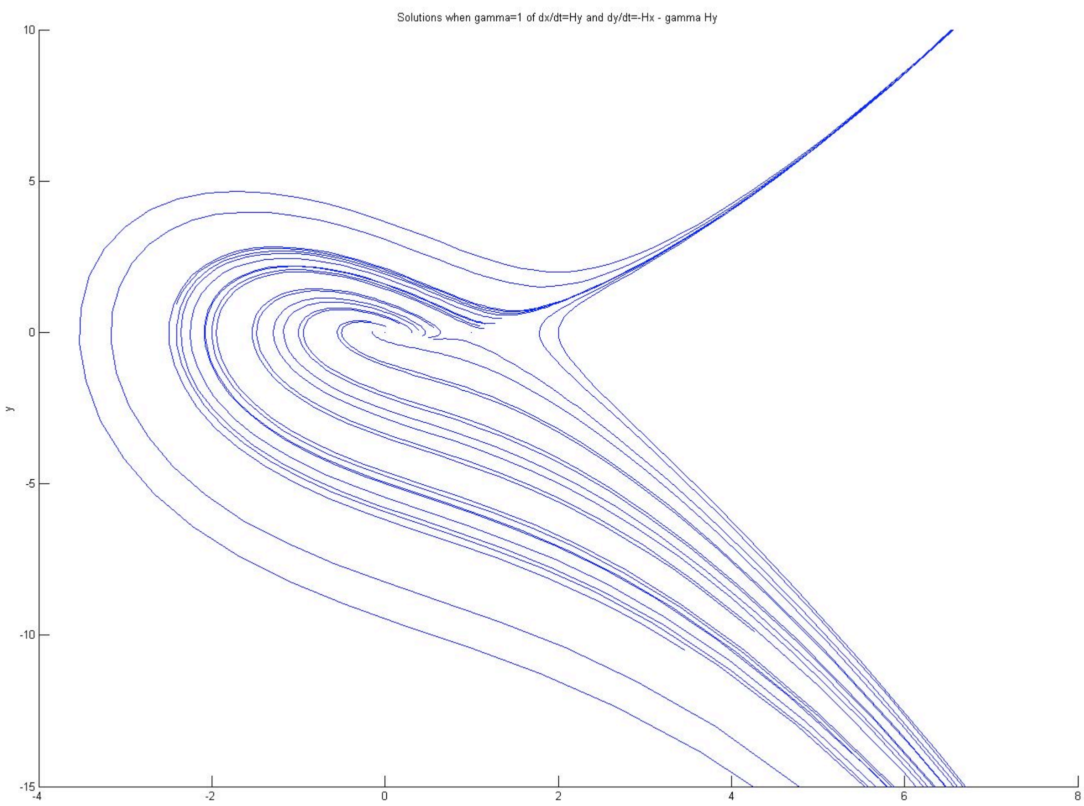
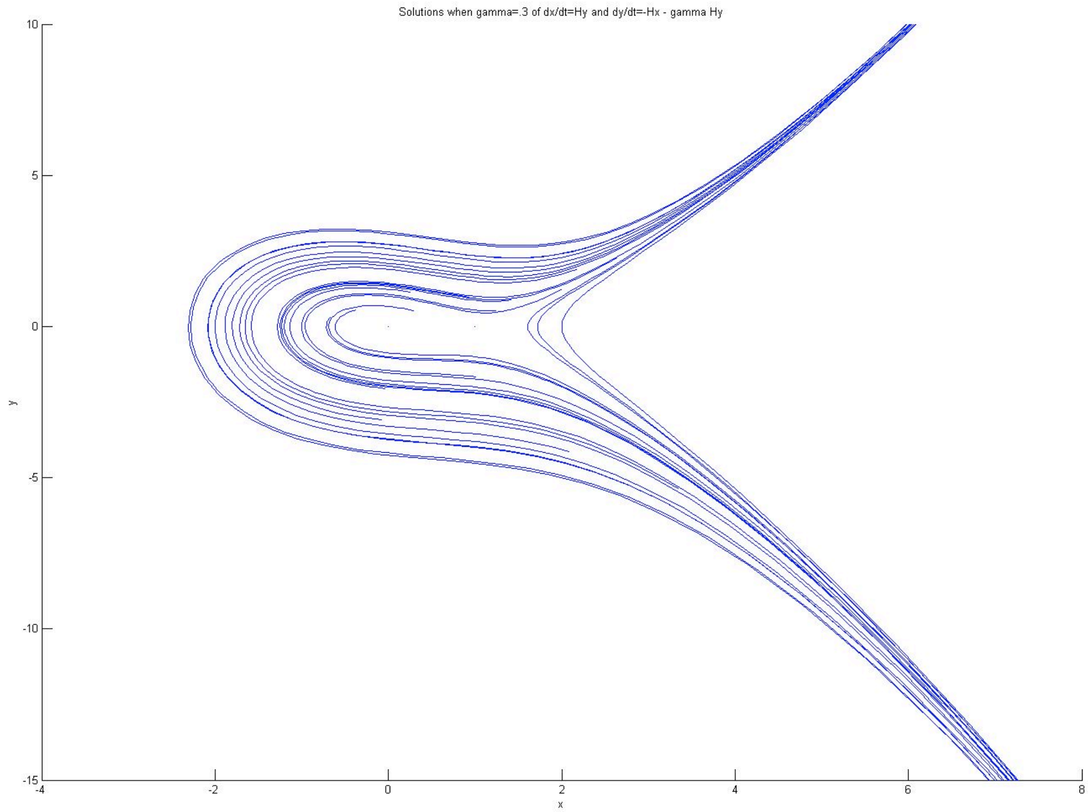
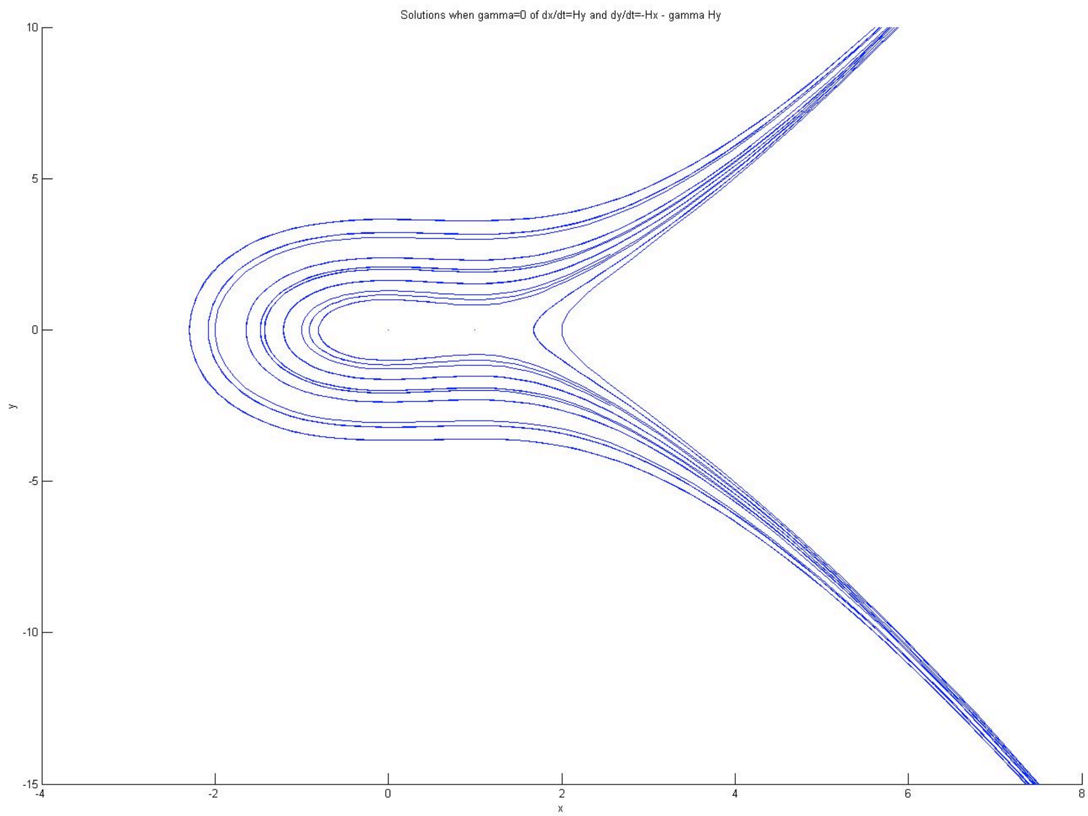
Graphs

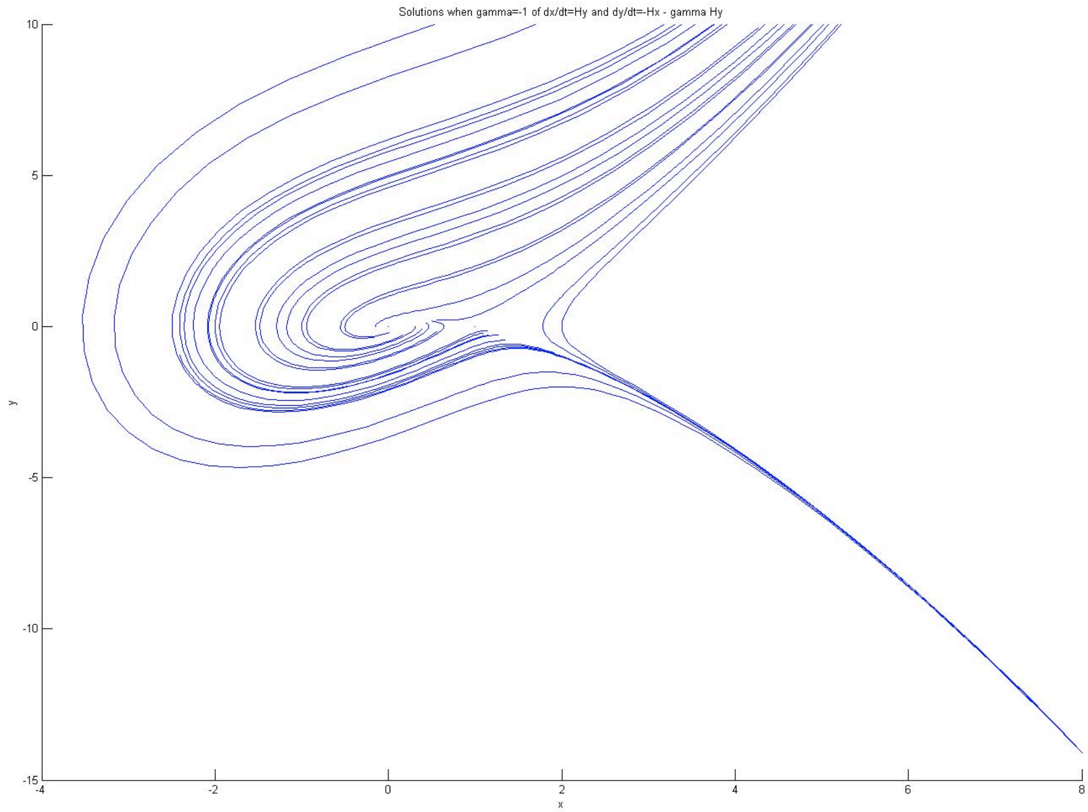
(Please zoom in to decipher the text of the titles)



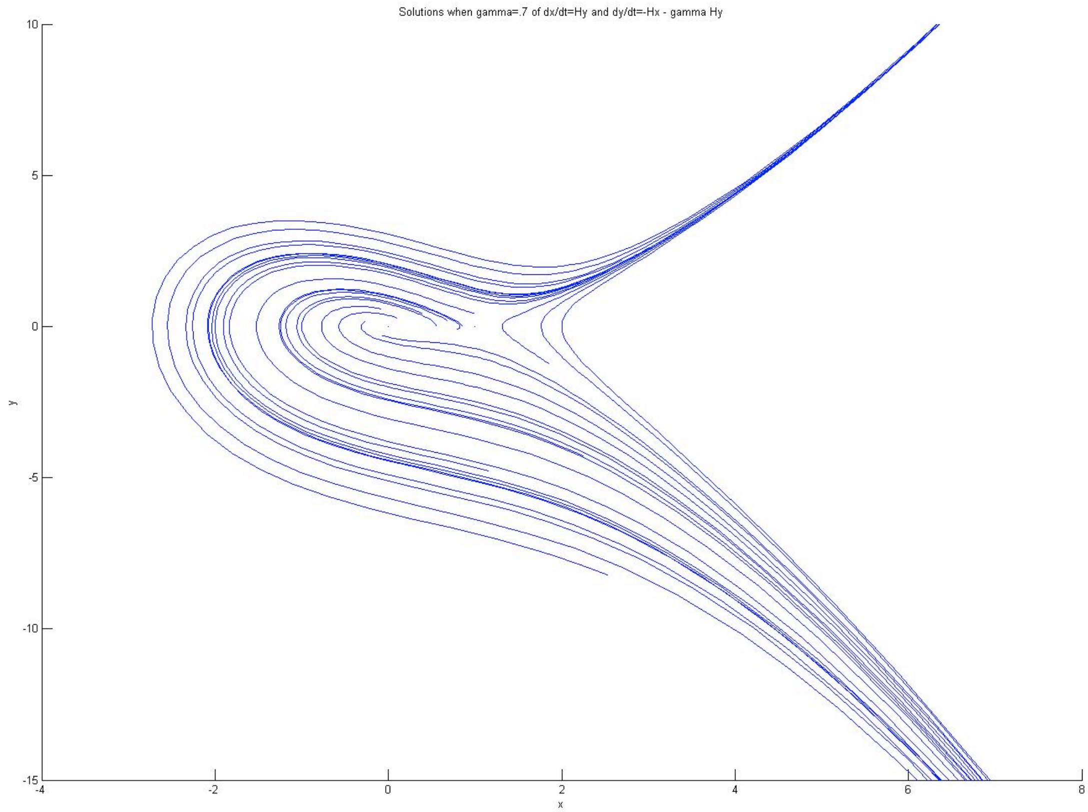


Solutions when gamma=.3 of dx/dt=Hy and dy/dt=-Hx - gamma Hy
10
5
0
-5
-10
-15
y
-4
-2
0
2
4
6
8
x


Solutions when gamma=0 of dx/dt=Hy and dy/dt=-Hx - gamma Hy
10
5
0
-5
-10
-15
y
-4
-2
0
2
4
6
8
x



Solutions when gamma=-1 of dx/dt=Hy and dy/dt=-Hx - gamma Hy
10
5
0
-5
-10
-15
y
-4
-2
0
2
4
6
8
x


Solutions when gamma=.7 of dx/dt=Hy and dy/dt=-Hx - gamma Hy
10
5
0
-5
-10
-15
y
-4
-2
0
2
4
6
8
x