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Damping a Pendulum

Equation: D2y+b*Dy+y=0

For the matrix:

x'=y

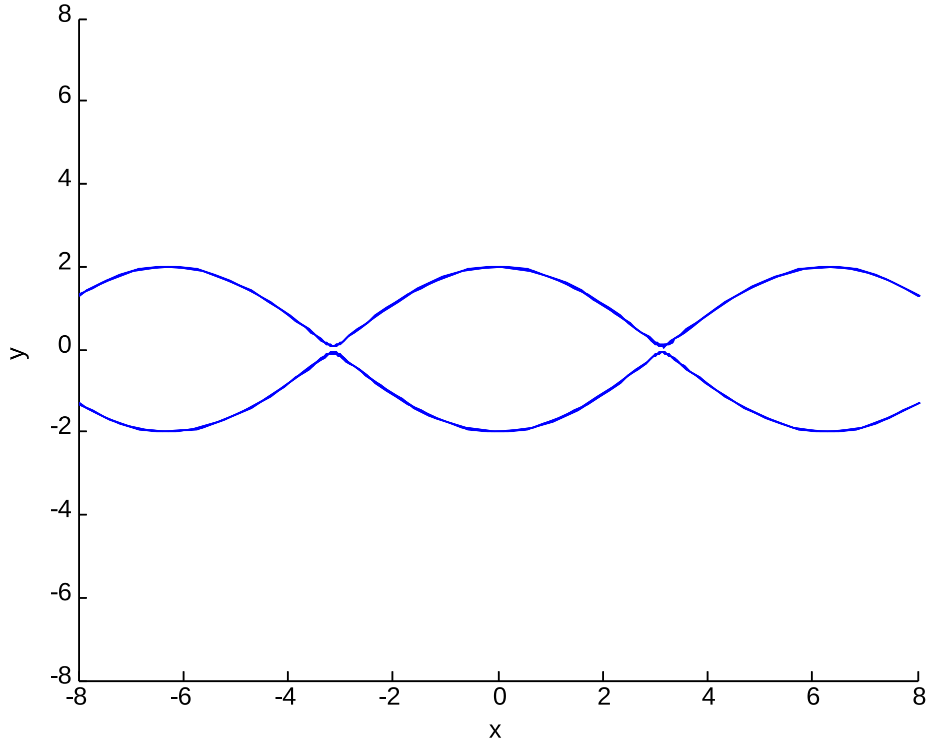
y'=-b*y-sin(x) where b is the damping constant

I used ODE45 to approximate the graph because there is no explicit solution

Code for pendulum pictures

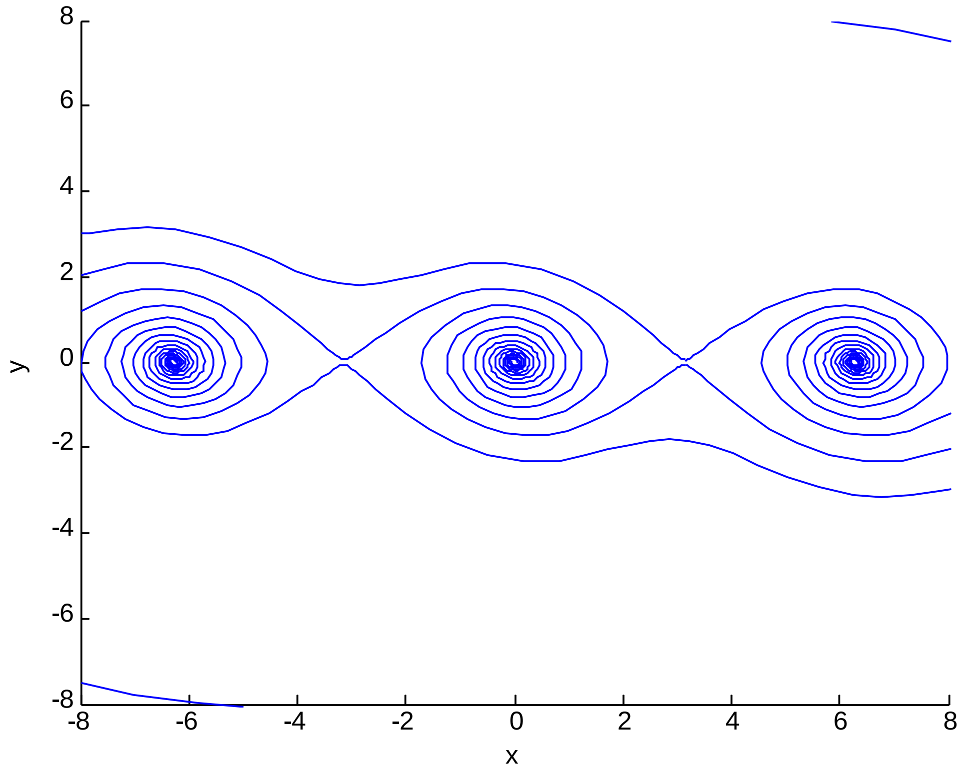
```
for i=1:numframes
c=(i-1)*0.15; xlabel 'x'; ylabel 'y'
axis ([-8 8 -8 8])
hold on;
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [-9 10]);
plot(xa(:,1),xa(:,2))
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [9 -10]);
plot(xa(:,1),xa(:,2))
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [pi 0.01]);
plot(xa(:,1),xa(:,2))
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [pi -0.05]);
plot(xa(:,1),xa(:,2))
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [-pi -0.05]);
plot(xa(:,1),xa(:,2))
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [-pi 0.05]);
plot(xa(:,1),xa(:,2))
f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 -50], [-pi 0.05]);
 plot(xa(:,1),xa(:,2))
```

```
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 -50], [-pi -0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 -50], [pi -0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 -50], [pi 0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [-3*pi 0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [-3*pi -0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [3*pi 0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 -50], [3*pi -0.05]);
 plot(xa(:,1),xa(:,2))
 f=@(t,x) [x(2); -sin(x(1))-c*x(2)];[t,xa]=ode45(f,[0 50], [3*pi -0.05]);
 plot(xa(:,1),xa(:,2))
hold off
figure
end
```



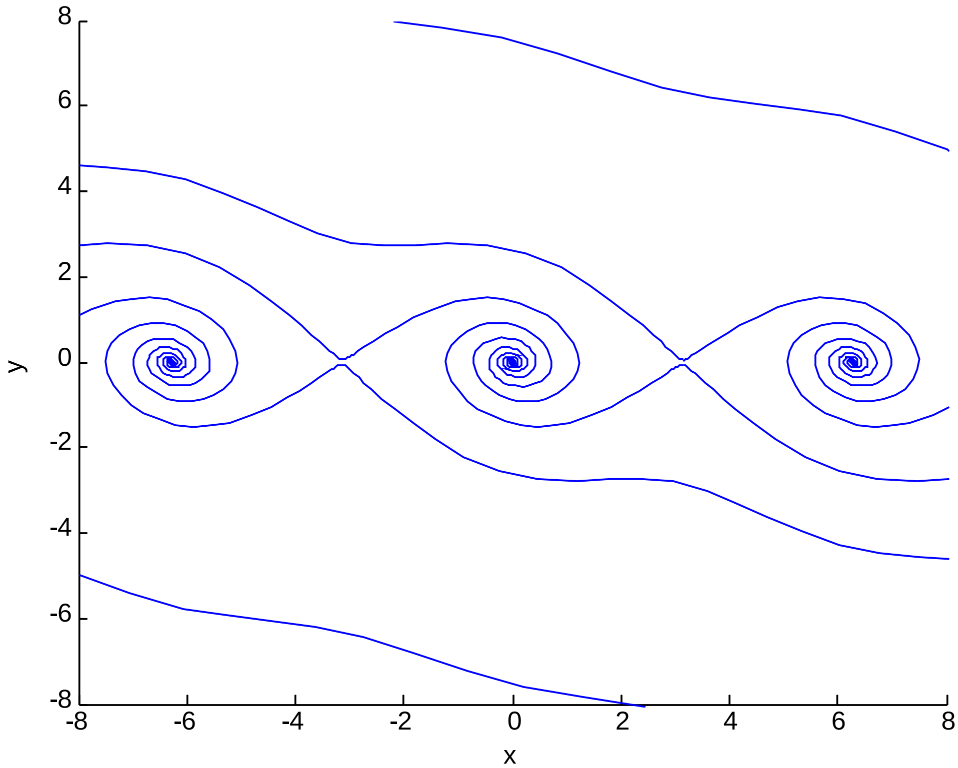
b=0

Here b=0, so the differential equation becomes D2y+y=0, which produces sinusoidal motion. This unlikely case is the only one which can be solved explicitly. It has critical points at n*pi , where n is any whole number.



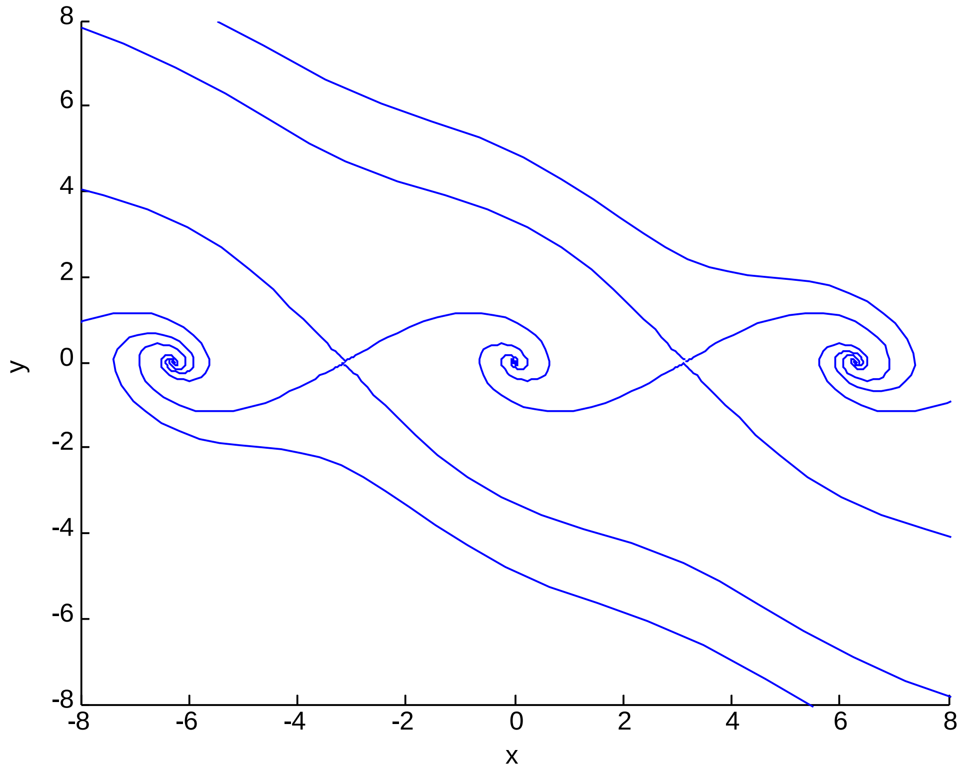
b=0.15

Here b is just a little bigger than 0, but one can easily see that, because b>0, spiral sinks , and saddles form. This is because the pendulum is under damped. The saddle points are where the pendulum is at the very top of the swing, so it is in an unstable equilibrium. The spiral sinks, whose critical points represent the bottom of the swing, are where the pendulum is oscillating less and less approaching the critical points.



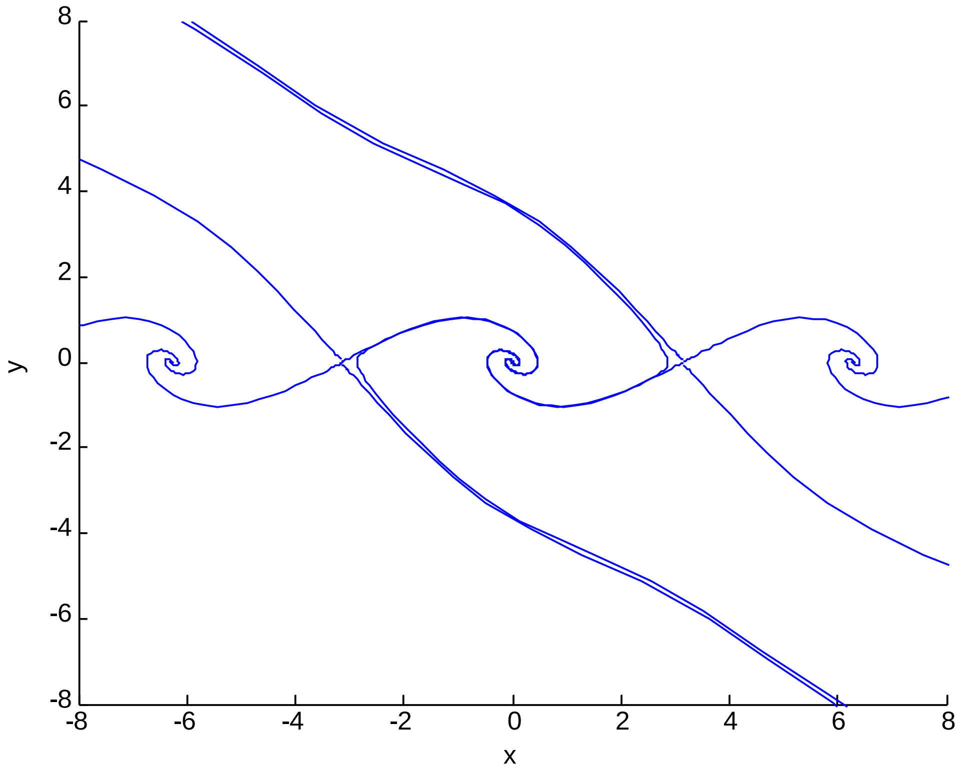
b=0.3

This portrait is again showing the phase portrait with spiral sinks at 2*n*pi, and saddle points at (n-1)*2*pi, where n is any whole number. This portrait solves the differential equation D2y+0.3*Dy+sin(y)=0 Here b is only slightly greater than 0, but now the oscillation is getting less and less, as b approaches the critical damping.



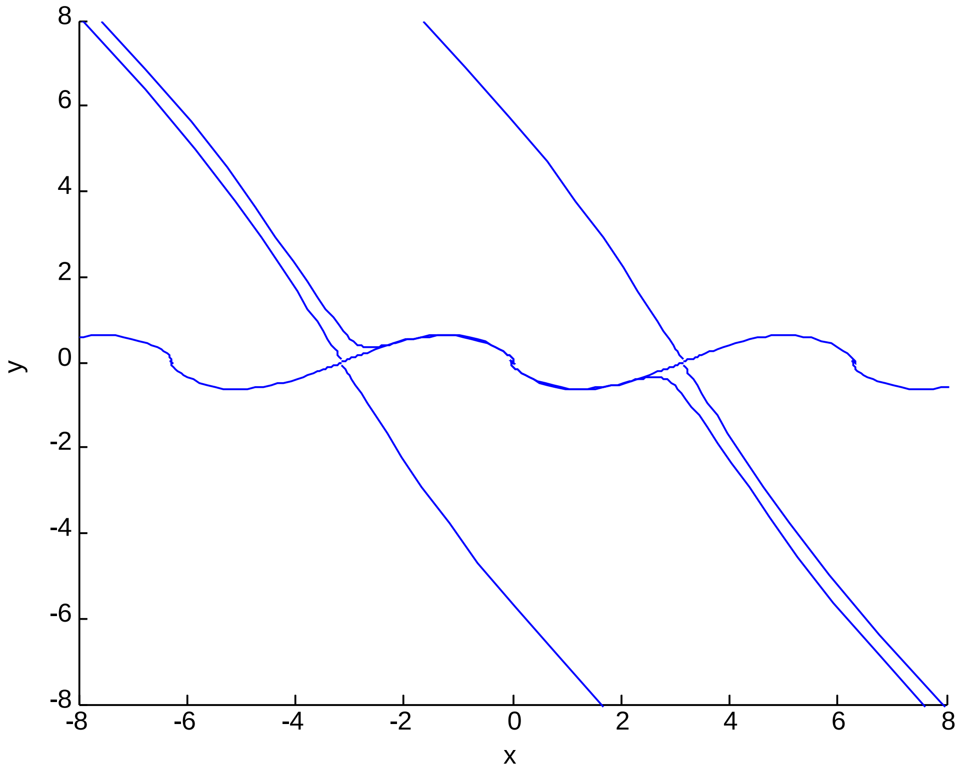
b=0.45

This portrait solves the differential equation D2y+0.45*Dy+sin(y)=0. It still keeps its critical points at n*pi, because sin(y)=0 at n*pi. Here b is getting even larger, and so the swing of the pendulum (the rate that the spiral sinks fall inward) is further increasing.



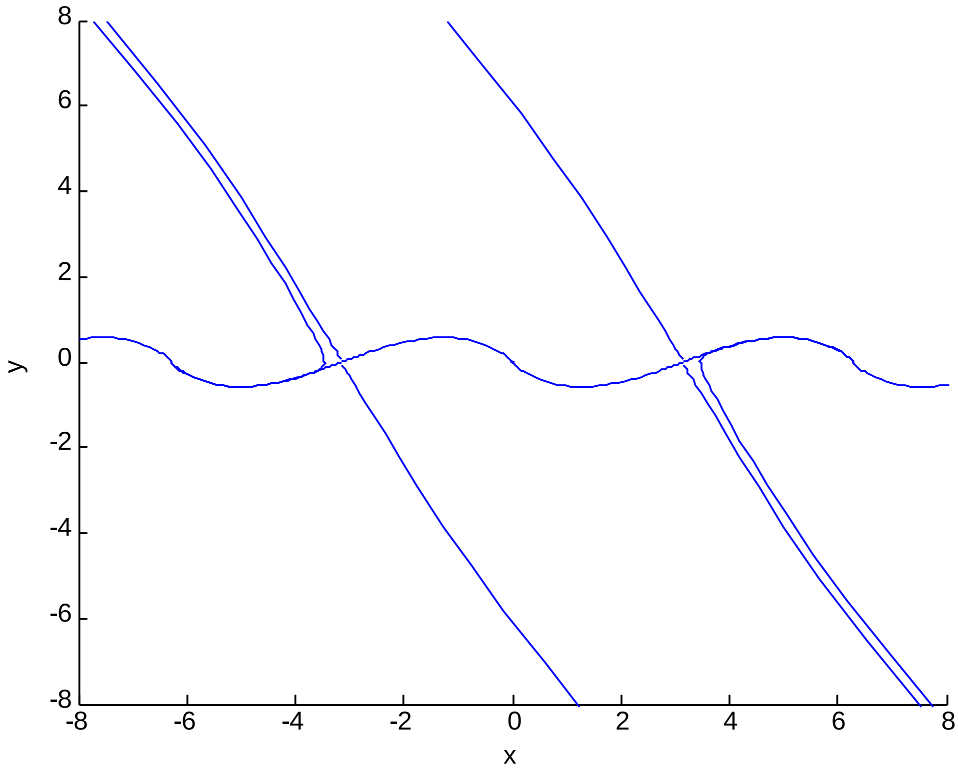
b=0.6

Now, the initial values are falling inward to the center, because b is increasing. The critical points are at n*pi, where all even Ns are spiral sinks (where the pendulum approaches the bottom of the swing) and all odd Ns are saddle points (where the pendulum is at the top of the swing)



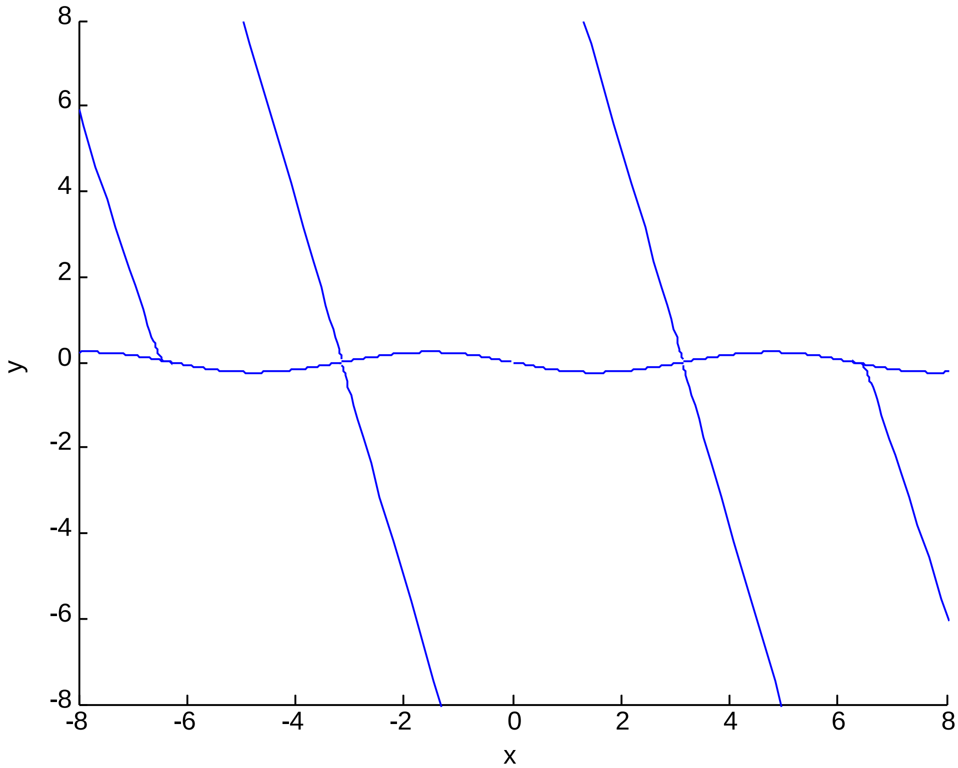
b=0.875

Here b is almost 1, so the portrait is almost like a real valued portrait, because the complex part of the eigen values are very nearly 0. The critical points are again at n*pi, with the spiral sinks at the odd n whole numbers.



b>1

Now, we have exceeded the critical damping point, so the eigen values now have no complex part. This means that the pendulum doesn't swing anymore. Instead it slowly approaches the critical points 2*n*pi, or stays at the top of the pendulum swing (at (n-1)*2*pi)



b>>1

This is what the pendulum is like if b is very large. There is not even a hint of swinging, and the points that approach the bottom of the pendulum swing (2*n*pi) are not essentially parallel to the lines coming out of the saddle points.

At first, when b=0 (i.e. pendulum is "frictionless"), there are no spiral sinks. Instead the phase portrait is positive and negative cosine graphs superimposed on each other, with critical points at (y=0, x=n*pi), where n is any whole number. The critical points are where the pendulum is on the bottom (at n*2*pi) or is balancing at the top of the circle (at (n-1)*2*pi). As b increases, the pendulum falls into the "wells," which are the spiral sinks, at a faster and faster pace (i.e. the slope at the initial points gets steeper and steeper). When the graph reaches critical damping, i.e. when sqrt(b^2-1) =0 at points very close to the critical points, than the plot looses the spiral sinks, because the eigen values only have real roots. From there, we can see that the equilibrium points at n*2*pi become nodal points, and the phase portrait of the areas around them become nodal sinks, because there is no oscillation. The areas around the "top of the pendulum" stay saddle points because a slight nudge pushes them to one n*2*pi valued critical points.