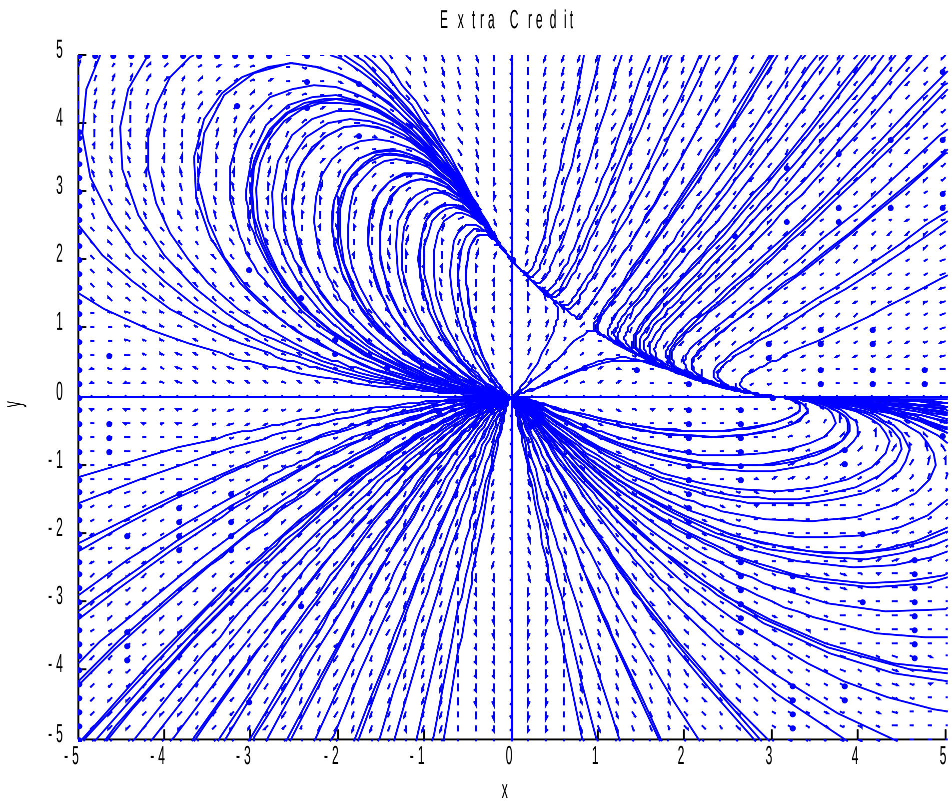
Jonathan Kumi's Extra Credit Project

When a = 0

```
syms x y
sys1 = x.*(1.5-0.5*x-y);
sys2 = y.*(2-y-1.125*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points:');disp([xc yc])

%Critical points are (0,0) (0,2) (3,0) (4/5 11/10)

A= jacobian([sys1 sys2], [x y])
evals = eig(A)

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (0,2):');
disp(double(subs(evals, {x, y}, {0, 2})))
disp('Eigenvalues at (3,0):');
disp(double(subs(evals, {x, y}, {3, 0})))
disp('Eigenvalues at (4/5, 11/10):');
disp(double(subs(evals, {x, y}, {4/5, 11/10})))

for a=0
  figure; hold on

  k=@(t,x)[(1-a)*(x(1)*(1.5-0.5*x(1)-x(2))+a*(x(1)*(1-0.5*x(1)-0.5*x(2)))); (1-a)*(x(2)*(2-
x(2)-1.125*x(1)))+a*(x(2)*(-0.25+0.5*x(1)))];
      for a=-3:.5:3
       for b=-3:.5:3
         [t,xa]=ode45(k,[0 10],[a b]);
         plot(xa(:,1),xa(:,2),'b');
         [t,xa]=ode45(k,[0 -10],[a b]);
         plot(xa(:,1),xa(:,2),'b');
       end
      end


   axis([-5 5 -5 5])
   title 'Extra Credit'
   xlabel 'x'
   ylabel 'y'
end
hold on
quiver(X, Y, U./L, V./L, 0.4)
axis([-5 5 -5 5])
```

% This function has 4 total critical points when a = 0. When the critical
% point is (0,0), the eigenvalues are (2,1.5) which makes the critical point an unstable nodal
%source. When the critical point is (0,2), the eigenvalues are (-0.5, -2.0) which
% makes the critical point a stable nodal sink. When the critical point is
% (3,0), the eigenvalues are (-1.3750,-1.5) which makes the critical point
% a stable nodal sink. Lastly, when the critical points are (4/5, 11/10),
% the eigenvalues are (0.3048, -1.8048) which makes the critical point an
% unstable saddle.

```
A =
[     3/2-x-y,          -x]
[      -9/8*y, 2-2*y-9/8*x]

evals =
 -17/16*x+7/4-3/2*y+1/16*(x^2-8*x+304*x*y+16-64*y+64*y^2)^(1/2)
 -17/16*x+7/4-3/2*y-1/16*(x^2-8*x+304*x*y+16-64*y+64*y^2)^(1/2)

Eigenvalues at (0,0):
    2.0000
 1.5000
Eigenvalues at (0,2):
   -0.5000
   -2.0000
Eigenvalues at (3,0):
   -1.3750
   -1.5000
Eigenvalues at (4/5, 11/10):
    0.3048
   -1.8048
```



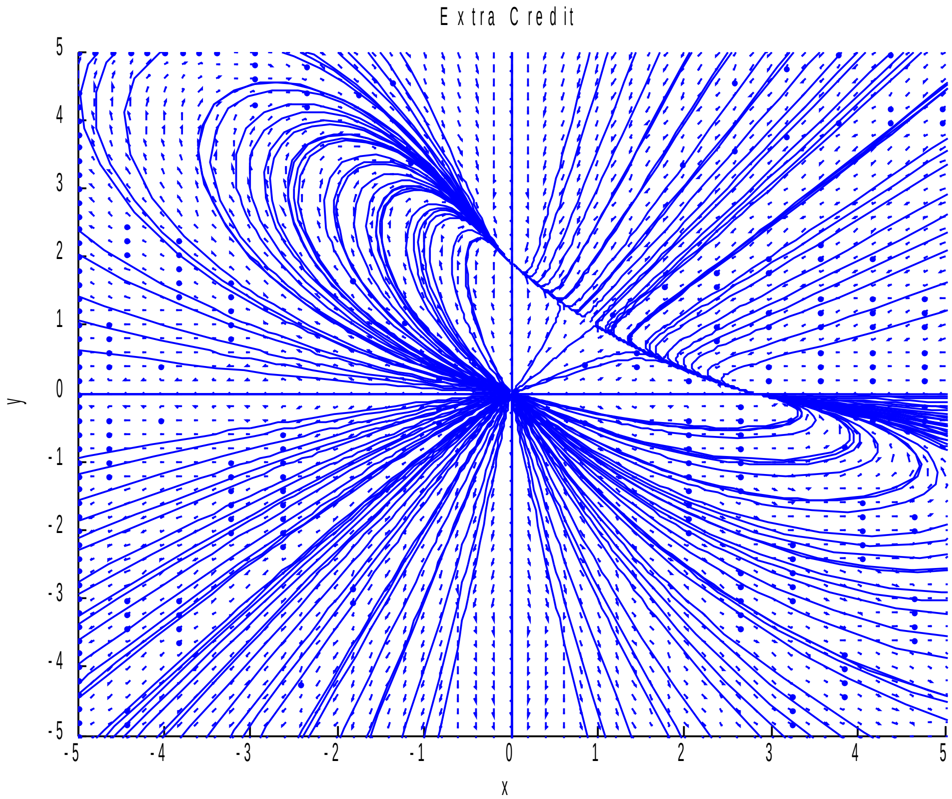
When a = 0.25

```
syms x y
sys1 = .75*x.*(1.5-0.5*x-y)+ .25*x.*(1-0.5*x-0.5*y);
sys2 = .75*y.*(2-y-1.125*x)+ .25*y.*(-.25+0.5*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points:');disp([xc yc])

%critical points are (0,0) (0, 23/12) (11/4,0) (58/65,69/65)

A= jacobian([sys1 sys2], [x y])
evals = eig(A)

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (0,23/12):');
disp(double(subs(evals, {x, y}, {0, 23/12})))
disp('Eigenvalues at (11/4,0):');
disp(double(subs(evals, {x, y}, {11/4, 0})))
disp('Eigenvalues at (58/65, 69/65):');
disp(double(subs(evals, {x, y}, {58/65, 69/65})))
```

for a=0.25

```
   figure; hold on

   k=@(t,x)[(1-a)*(x(1)*(1.5-0.5*x(1)-x(2))+a*(x(1)*(1-0.5*x(1)-0.5*x(2)))); (1-a)*(x(2)*(2-
x(2)-1.125*x(1)))+a*(x(2)*(-0.25+0.5*x(1)))];
       for a=-3:.5:3
        for b=-3:.5:3
           [t,xa]=ode45(k,[0 10],[a b]);
           plot(xa(:,1),xa(:,2),'b');
           [t,xa]=ode45(k,[0 -10],[a b]);
           plot(xa(:,1),xa(:,2),'b');
        end
       end


    axis([-5 5 -5 5])
    title 'Extra Credit'
    xlabel 'x'
    ylabel 'y'
end
hold on
quiver(X, Y, U./L, V./L, 0.4)
axis([-5 5 -5 5])

%When a=.25, the function still has 4 critical points. When the critical
%points are (0,0), the eigenvalues are (1.4375, 1.3750) which makes the
%critical point an unstable nodal source. The critical point at (0,0)
%remained an unstable source from a=0 to a=.25. When the critical
%points are (0,23/12), the eigenvalues are (-0.3021, -1.4375) which makes the
%critical point a stable nodal sink. When the critical
%points are (11/14,0), the eigenvalues are (-0.5391, -1.3750) which makes the
%critical point an stable nodal sink. When the critical
%points are (58/65,69/65), the eigenvalues are (0.1703,-1.4126) which makes the
%critical point an unstable saddle. The function changed from a=0 to a=.25
%but the change isn't too significant when both phase portraits are compared.

A =

[       11/8-x-7/8*y,             -7/8*x]
[           -23/32*y, 23/16-3/2*y-23/32*x]

evals =

 -55/64*x+45/32-19/16*y+1/64*(81*x^2+36*x+2216*x*y+4-80*y+400*y^2)^(1/2)
 -55/64*x+45/32-19/16*y-1/64*(81*x^2+36*x+2216*x*y+4-80*y+400*y^2)^(1/2)

Eigenvalues at (0,0):
```

1.4375
1.3750
Eigenvalues at (0,23/12):
-0.3021
-1.4375
Eigenvalues at (11/4,0):
-0.5391
-1.3750
Eigenvalues at (58/65, 69/65):
0.1703
-1.4126



When a=.50

```
syms x y
sys1 = .50*x.*(1.5-0.5*x-y)+ .50*x.*(1-0.5*x-0.5*y);
sys2 = .50*y.*(2-y-1.125*x)+ .50*y.*(-.25+0.5*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points:');disp([xc yc])

%critical points are (0,0) (0, 7/4) (5/2,0) (-2,3)

A= jacobian([sys1 sys2], [x y])
evals = eig(A)
```

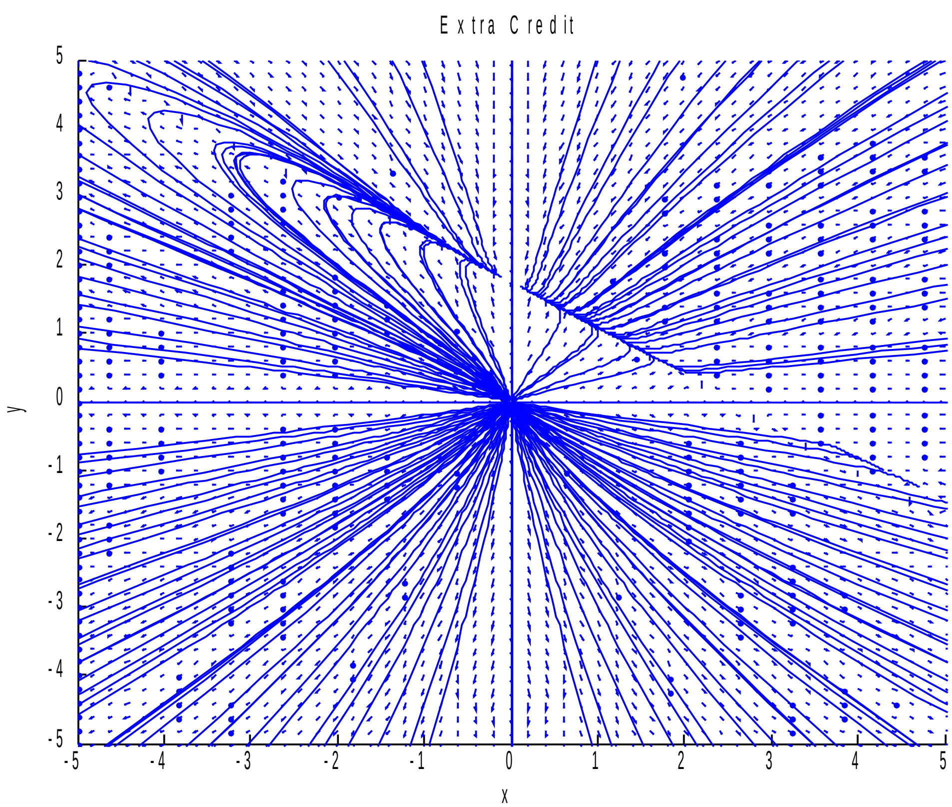
```
disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (0,7/4):');
disp(double(subs(evals, {x, y}, {0, 7/4})))
disp('Eigenvalues at (5/2,0):');
disp(double(subs(evals, {x, y}, {5/2, 0})))
disp('Eigenvalues at (-2, 3):');
disp(double(subs(evals, {x, y}, {-2,3})))


for a=0.50
  figure; hold on

  k=@(t,x)[(1-a)*(x(1)*(1.5-0.5*x(1)-x(2))+a*(x(1)*(1-0.5*x(1)-0.5*x(2)))); (1-a)*(x(2)*(2-
x(2)-1.125*x(1)))+a*(x(2)*(-0.25+0.5*x(1)))];
     for a=-3:.5:3
      for b=-3:.5:3
        [t,xa]=ode45(k,[0 10],[a b]);
        plot(xa(:,1),xa(:,2),'b');
        [t,xa]=ode45(k,[0 -10],[a b]);
        plot(xa(:,1),xa(:,2),'b');
      end
     end


   axis([-5 5 -5 5])
   title 'Extra Credit'
   xlabel 'x'
   ylabel 'y'
end
hold on
quiver(X, Y, U./L, V./L, 0.4)
axis([-5 5 -5 5])

 %When a=.50, the function begins to show significant change even though it still has 4 critical
%points. When the critical points are (0,0), the eigenvalues are (1.25, 0.8750) which makes the
%critical point an unstable nodal source. The critical point at (0,0)
%remained an unstable source from a=0 to a=.50. When the critical
%points are (0,7/4), the eigenvalues are (-0.0625, -0.8750) which makes the
%critical point a stable nodal sink. When the critical
%points are (5/2,0), the eigenvalues are (0.0938, -1.25) which makes the
%critical point an unstable saddle. When the critical
%points are (-2,3), the eigenvalues are (0.1453,-0.6453) which makes the
%critical point an unstable saddle. The function changed from a=.25 to
%a=.50 because another critical point became an unstable saddle. This
%difference can be detected by looking at the phase portrait as well as the
```

%directional field.
A =
[5/4-x-3/4*y,   -3/4*x]
[-5/16*y, 7/8-y-5/16*x]

evals =

-21/32*x+17/16-7/8*y+1/32*(121*x^2-132*x+152*x*y+36+48*y+16*y^2)^(1/2)
-21/32*x+17/16-7/8*y-1/32*(121*x^2-132*x+152*x*y+36+48*y+16*y^2)^(1/2)

Eigenvalues at (0,0):
1.2500
0.8750
Eigenvalues at (0,7/4):
-0.0625
-0.8750
Eigenvalues at (5/2,0):
0.0938
-1.2500
Eigenvalues at (-2, 3):
0.1453
-0.6453

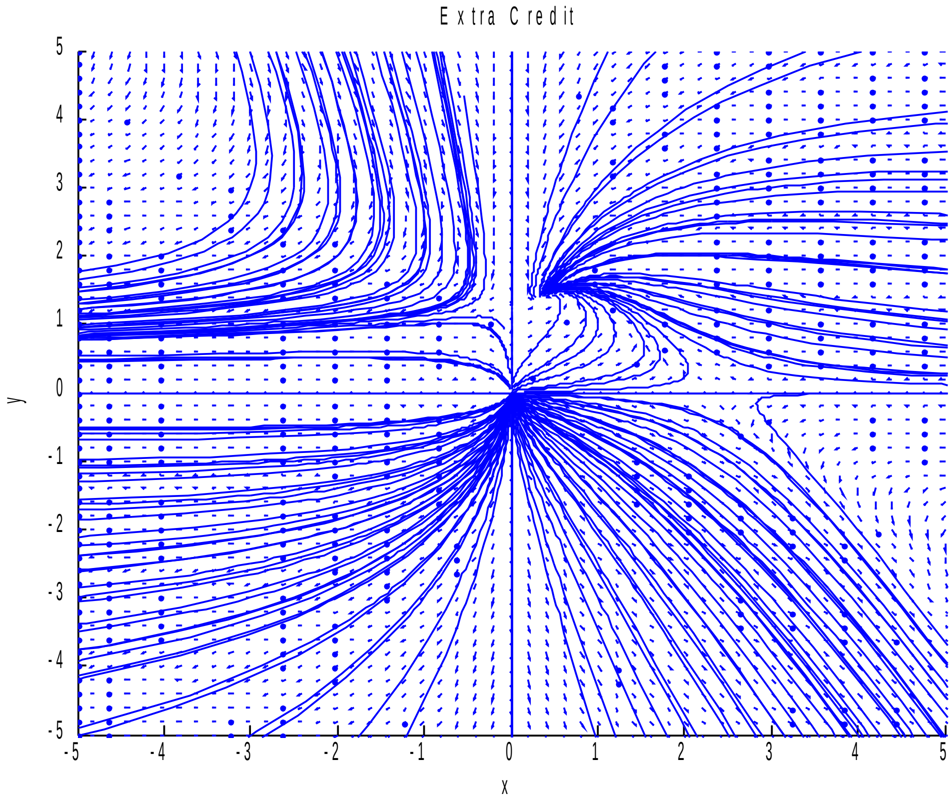


when a = 0.75

```
syms x y
sys1 = .25*x.*(1.5-0.5*x-y)+ .75*x.*(1-0.5*x-0.5*y);
sys2 = .25*y.*(2-y-1.125*x)+ .75*y.*(-.25+0.5*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points:');disp([xc yc])

%critical points are (0,0) (0, 5/4) (9/4,0) (22/47,67/47)

A= jacobian([sys1 sys2], [x y])
evals = eig(A)

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (0,5/4):');
disp(double(subs(evals, {x, y}, {0, 5/4})))
disp('Eigenvalues at (9/4,0):');
disp(double(subs(evals, {x, y}, {9/4, 0})))
disp('Eigenvalues at (22/47, 67/47):');
disp(double(subs(evals, {x, y}, {22/47,67/47})))


for a=0.75
  figure; hold on

  k=@(t,x)[(1-a)*(x(1)*(1.5-0.5*x(1)-x(2))+a*(x(1)*(1-0.5*x(1)-0.5*x(2)))); (1-a)*(x(2)*(2-
x(2)-1.125*x(1)))+a*(x(2)*(-0.25+0.5*x(1)))];
      for a=-3:.5:3
       for b=-3:.5:3
         [t,xa]=ode45(k,[0 10],[a b]);
         plot(xa(:,1),xa(:,2),'b');
         [t,xa]=ode45(k,[0 -10],[a b]);
         plot(xa(:,1),xa(:,2),'b');
       end
      end


   axis([-5 5 -5 5])
   title 'Extra Credit'
   xlabel 'x'
   ylabel 'y'
end
hold on
quiver(X, Y, U./L, V./L, 0.4)
axis([-5 5 -5 5])
```

%When a=.75, the function continues to show significant change but still has 4 critical points.
%When the critical points are (0,0), the eigenvalues are (1.1250, 0.3125) which makes the
%critical point an unstable nodal source. The critical point at (0,0)
%remained an unstable source from a=0 to a=.75. When the critical
%points are (0,5/4), the eigenvalues are (0.3438, -0.3125) which makes the
%critical point an unstable saddle point. When the critical
%points are (9/4,0), the eigenvalues are (0.5234, -1.1250) which makes the
%critical point an unstable saddle. When the critical
%points are (22/47,67/47), the eigenvalues are (-0.2952+0.1880i, -0.2952-0.1880i) which
%makes the critical point a spiral source. The eigenvalues became complex. The function
%changed from a=.50 to a=.75 because a critical point became a clockwise spiral %source.
%This difference can be detected by looking at the phase portrait as well as the
% directional field once again.

```
A =

[      9/8-x-5/8*y,           -5/8*x]
[          3/32*y, 5/16-1/2*y+3/32*x]



evals =

 -29/64*x+23/32-9/16*y+1/64*(1225*x^2-1820*x+40*x*y+676-208*y+16*y^2)^(1/2)
 -29/64*x+23/32-9/16*y-1/64*(1225*x^2-1820*x+40*x*y+676-208*y+16*y^2)^(1/2)


Eigenvalues at (0,0):
    1.1250
    0.3125

Eigenvalues at (0,5/4):
    0.3438
   -0.3125

Eigenvalues at (9/4,0):
    0.5234
   -1.1250

Eigenvalues at (22/47, 67/47):
  -0.2952 + 0.1880i
  -0.2952 - 0.1880i
```



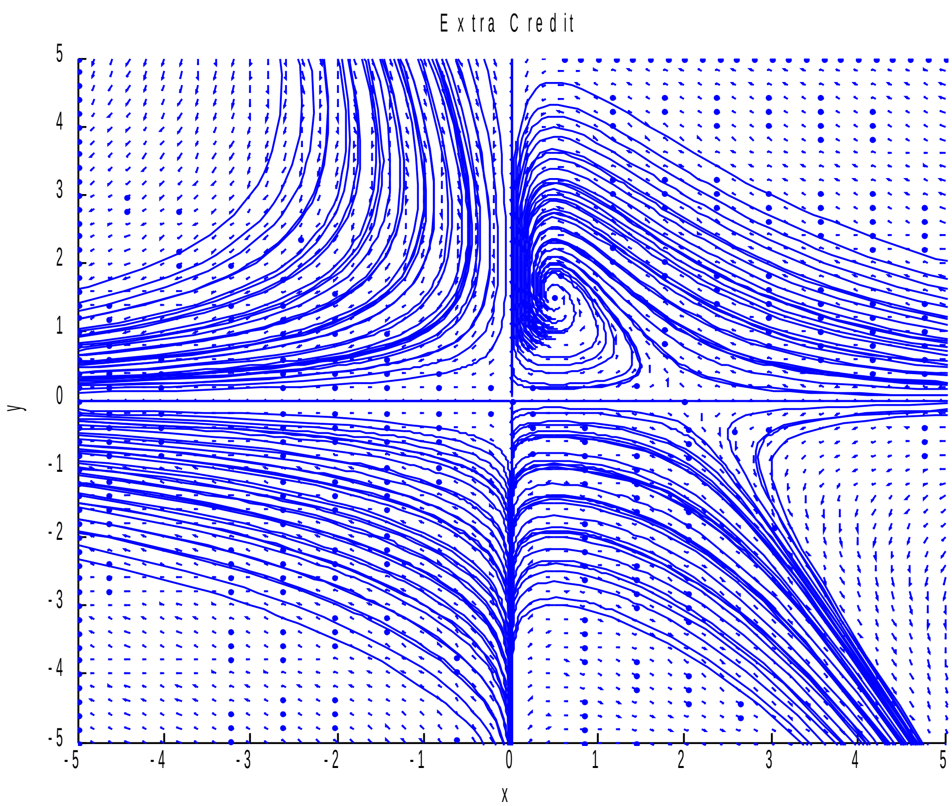
when a = 1.00

```
syms x y
sys1 = x.*(1-0.5*x-0.5*y);
sys2 = y.*(-.25+0.5*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points:');disp([xc yc])

%critical points are (0,0) (2, 0) (1/2,3/2)

A= jacobian([sys1 sys2], [x y])
evals = eig(A)

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x, y}, {0, 0})))
disp('Eigenvalues at (2,0):');
disp(double(subs(evals, {x, y}, {2, 0})))
disp('Eigenvalues at (1/2,3/2):');
disp(double(subs(evals, {x, y}, {1/2, 3/2})))
```

```
for a=1
   figure; hold on

   k=@(t,x)[((1-a)*(x(1)*(1.5-0.5*x(1)-x(2))))+a*(x(1)*(1-0.5*x(1)-0.5*x(2))); ((1-a)*(x(2)*(2-
x(2)-1.125*x(1))))+a*(x(2)*(-0.25+0.5*x(1)))];
       for a=-3:.5:3
        for b=-3:.5:3
           [t,xa]=ode45(k,[0 10],[a b]);
           plot(xa(:,1),xa(:,2),'b');
           [t,xa]=ode45(k,[0 -10],[a b]);
           plot(xa(:,1),xa(:,2),'b');
        end
       end


    axis([-5 5 -5 5])
    title 'Extra Credit'
    xlabel 'x'
    ylabel 'y'
end
hold on
quiver(X, Y, U./L, V./L, 0.4)
axis([-5 5 -5 5])

%When a=1, the function continues to show significant change and only has 3 critical points.
%When the critical points are (0,0), the eigenvalues are (1, -0.25) which makes the
%critical point an unstable saddle. The critical point at (0,0)
%changed to an unstable saddle from an unstable source from a=.75 to a=1. When the critical
%points are (2,0), the eigenvalues are (0.75, -1) which makes the
%critical point an unstable saddle point. When the critical
%points are (1/2,3/2), the eigenvalues are (-0.1250 + 0.4146i, -0.1250- 0.4146i) which makes
%the critical point a spiral source. The function changed from a=.50 to
%a=.75 because a critical point became a clockwise spiral source once again. This
%difference can be detected by looking at the phase portrait as well as the
%directional field once again. All in all, for a=0 to a=1, the phase
%portraits show significant difference. The portraits all have unique
%critical points with their own eigenvalues which determine what the phase
%portraits look like.
A =
[  1-x-1/2*y,     -1/2*x]
[      1/2*y, -1/4+1/2*x]



evals =
```

3/8-1/4*x-1/4*y+1/8*(25-60*x-20*y+36*x^2+8*x*y+4*y^2)^(1/2)
3/8-1/4*x-1/4*y-1/8*(25-60*x-20*y+36*x^2+8*x*y+4*y^2)^(1/2)

Eigenvalues at (0,0):
1.0000
-0.2500
Eigenvalues at (2,0):
0.7500
-1.0000
Eigenvalues at (1/2,3/2):
-0.1250 + 0.4146i
-0.1250 - 0.4146i



<u>Conclusion</u>

This extra project assignment showed me how phase portraits change when alpha is used as an interval. The alpha value determined how the phase portraits shifted. For example, when a=0, the phase portrait looked way different than when a=1. Combining competing species equations with predator-prey equations really show how phase portraits can evolve. Throughout the project, critical points moved around and shifted to make different portraits. Some critical points even vanished whiles some changed from saddles to spirals. The basic story behind the project is that changing the value of alpha really affects the outcome of the phase portrait. In my case, I experimented with 5 different alpha values and produced 5 different phase portraits.