

Math 246 Extra Credit Projects, Spring 2009

Seung Lee

$$d/dt = (x; y) = -(H_y; H_x)$$

$$H(x, y) = -(1/2)y^2 + (1/4)x^4 + (1/2)cx^2$$

Problem: Using Matlab create the phase portraits for multiple cases of the variable, c , including a negative example, positive example, and zero. After doing so describe what happens as c changes.

$$H_y = y = -x(2)$$

$$H_x = x^3 + cx = x(1)^3 + c*x(1)$$

Case 1: c = -10

```
>> f = @(t, x) [x(2); -x(1)^3 + 10*x(1)];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

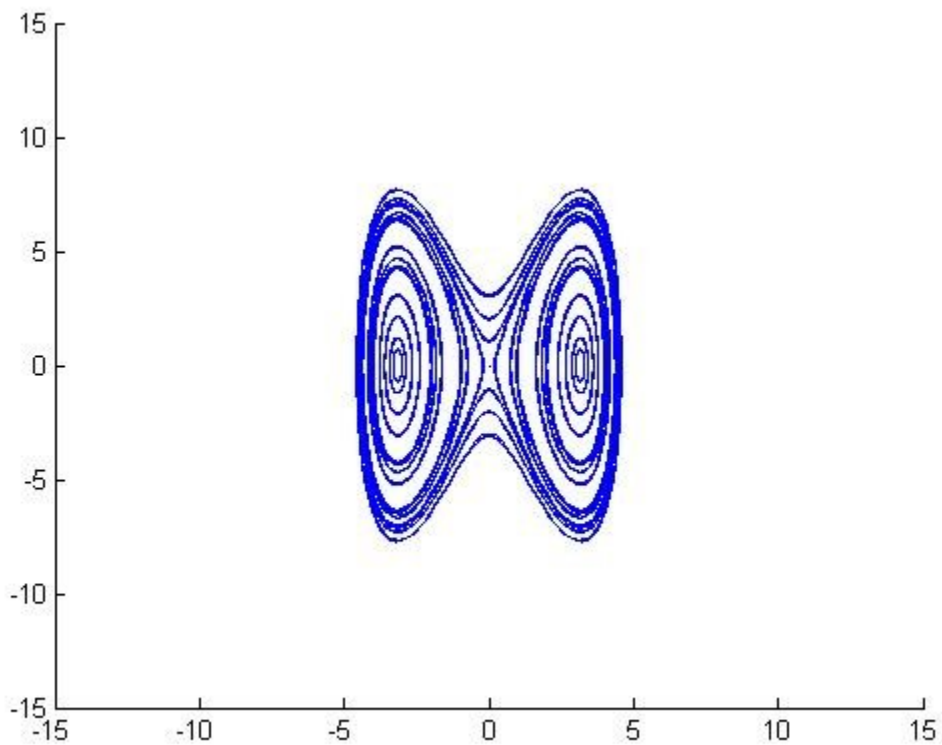
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
axis([-15 15 -15 15])
```



Case 2: c = -5

```
>> f = @(t, x) [x(2); -x(1)^3 + 5*x(1)];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

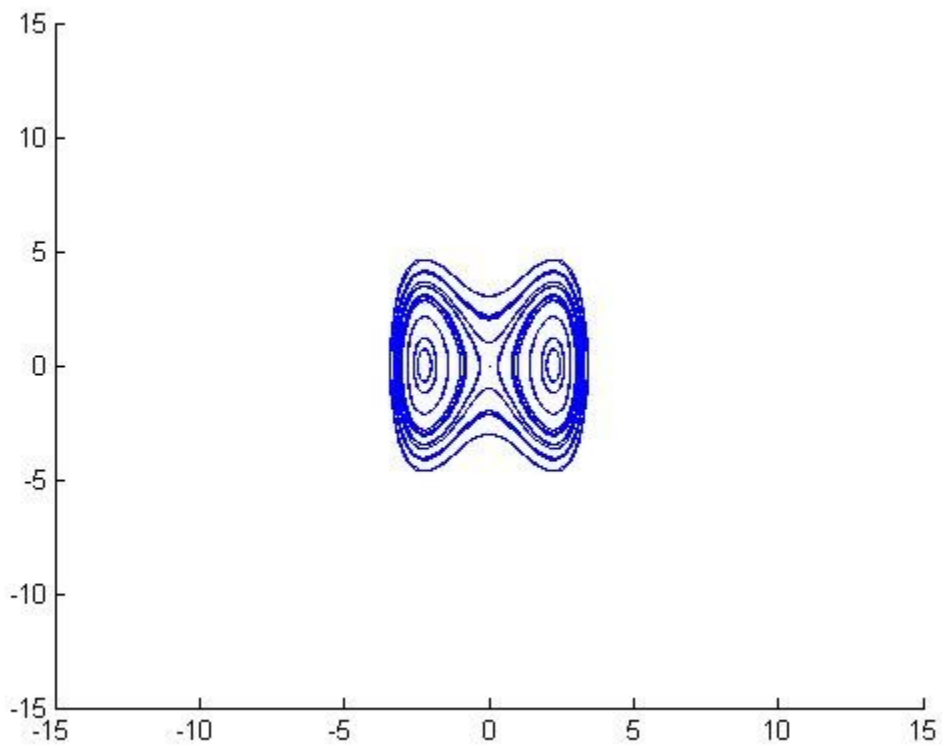
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
>> axis ([-15 15 -15 15])
```



Case 3: c = -1

```
>> f = @(t, x) [x(2); -x(1)^3 + x(1)];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

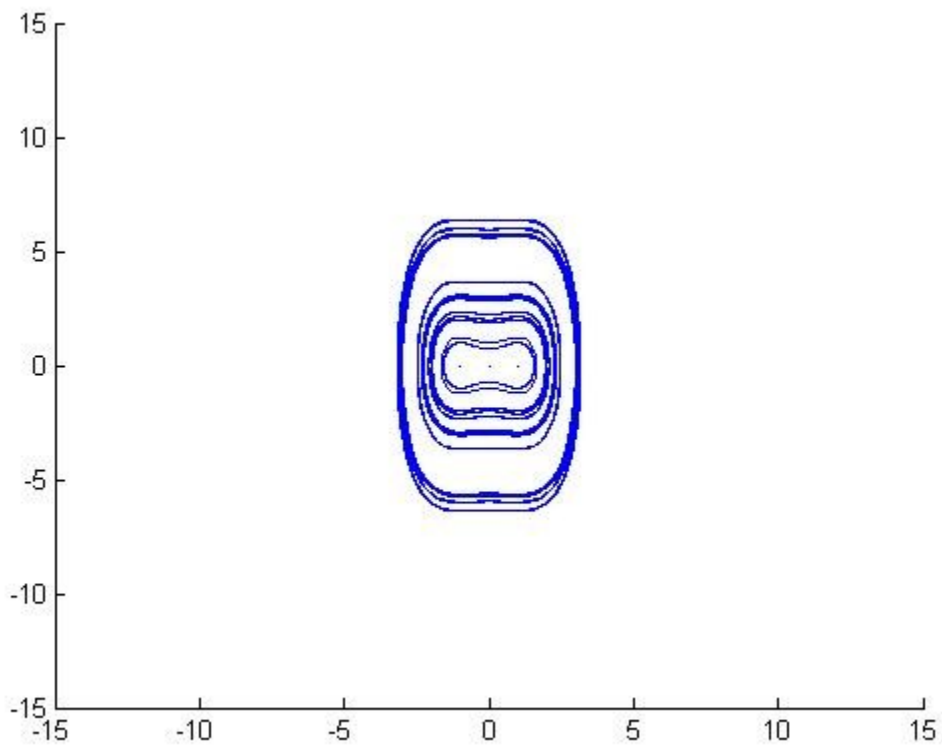
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
axis([-15 15 -15 15])
```



Case 4: $c = 0$

```
>> f = @(t, x) [x(2); -x(1)^3];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

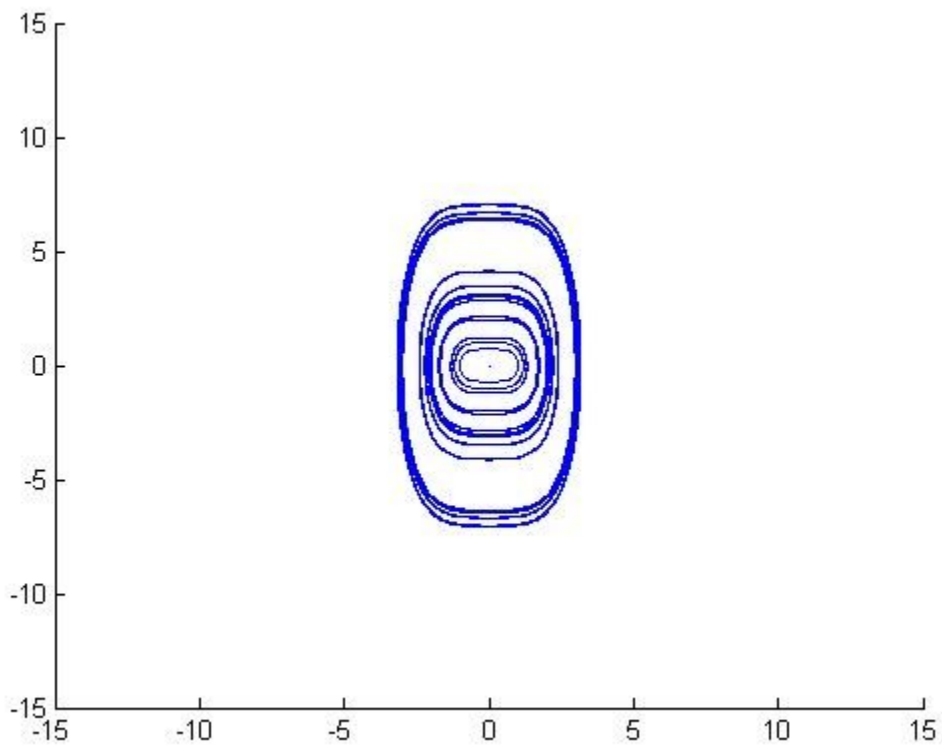
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
>> axis([-15 15 -15 15])
```



Case 5: $c = 1$

```
>> f = @(t, x) [x(2); -x(1)^3 - x(1)];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

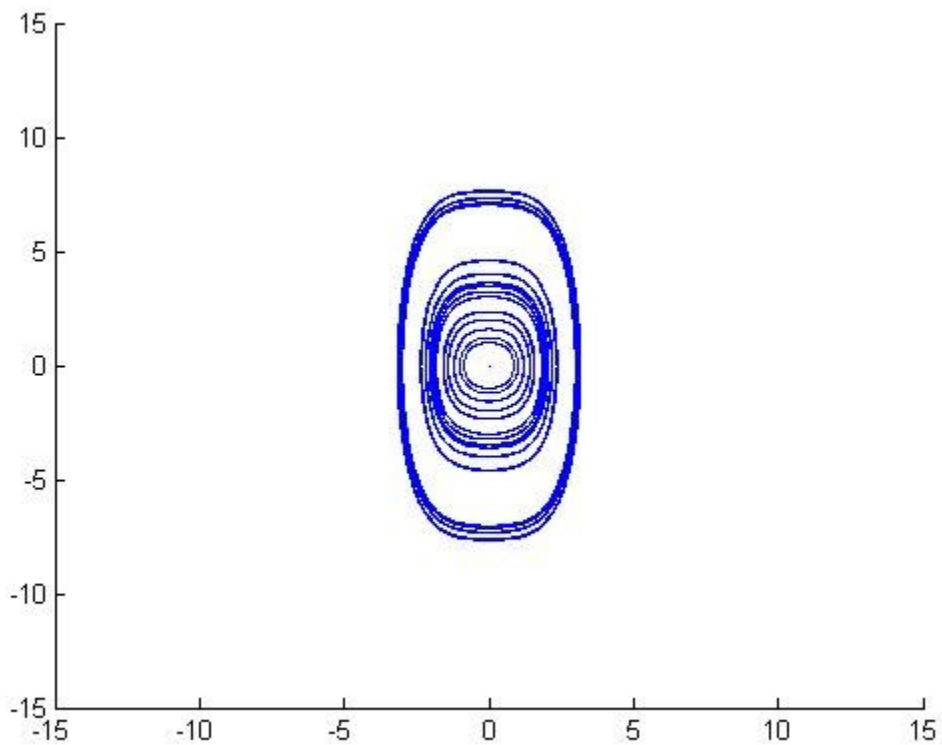
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
>> axis([-15 15 -15 15])
```



Case 6: $c = 5$

```
>> f = @(t, x) [x(2); -x(1)^3 - 5*x(1)];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

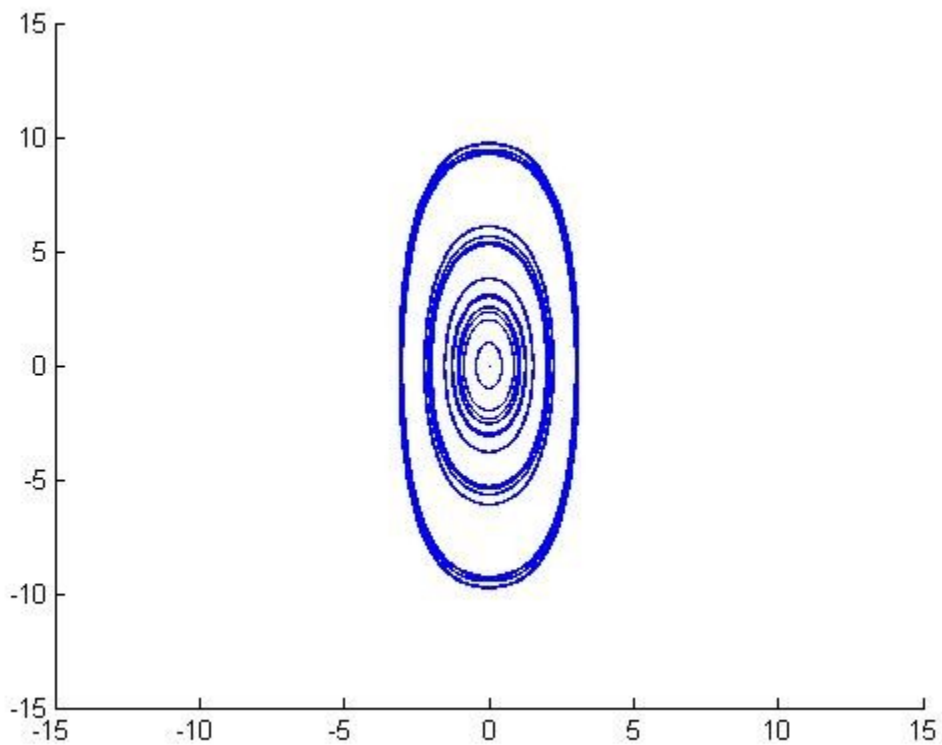
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
>> axis([-15 15 -15 15])
```



Case 7: c= 10

```
>> f = @(t, x) [x(2); -x(1)^3 - 10*x(1)];
```

```
>> figure, hold on
```

```
>> for a = -3:3
```

```
for b = -3:3
```

```
[t, xa] = ode45(f, [0 3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

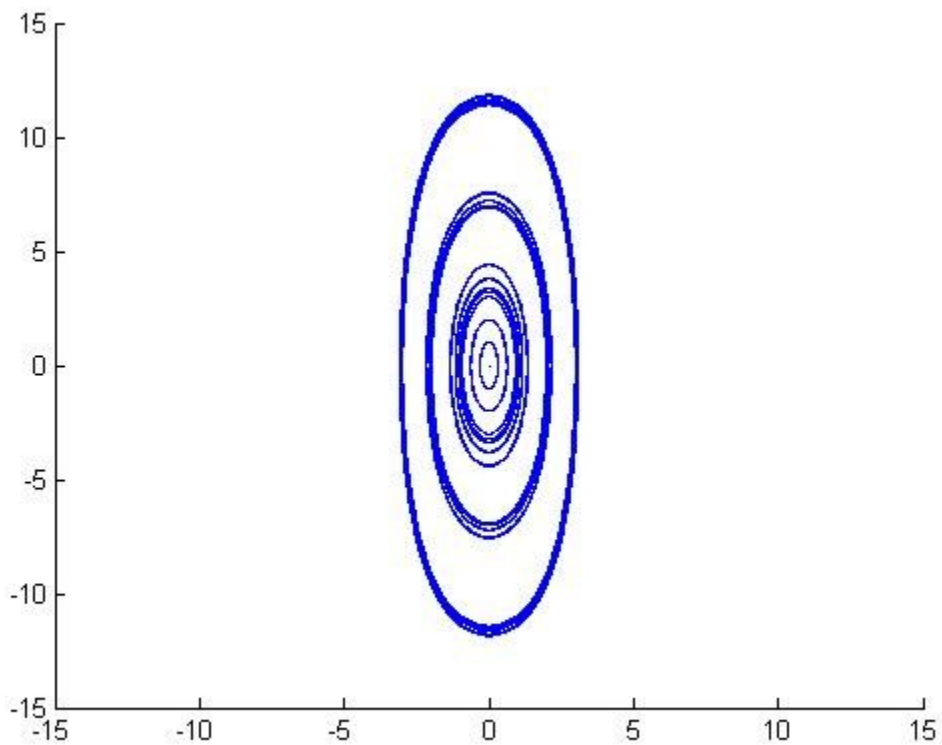
```
[t, xa] = ode45(f, [0 -3], [a b]);
```

```
plot(xa(:,1), xa(:,2))
```

```
end
```

```
end
```

```
>> axis ([-15 15 -15 15])
```



Analysis

The function shows very interesting behavior in its phase portraits as c changes.

In cases 1, 2, and 3 where the c is a negative number, the phase portraits show 3 critical points.

In all portraits the origin is always a critical point, but the two other ones shift inwards towards the origin. In all three figures the origin is an unstable saddle point, and the other two points are stable center. In case 4 the three critical points we have seen in the negative examples have merged into one at the origin. As c increases in the positive region shown through the cases numbered 5, 6, and 7 the phase portraits are shown as a center portrait, which keeps changing to look more and more like an ellipse as c increases.