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Matlab Extra Credit

>> syms x y q
H = 1/2*y^2+1/2*x^2-1/4*x^4
f = diff(H, y)-q*diff(H, x)
g = -diff(H, x)-q*diff(H, y)
[X Y] = solve(f,g); [X Y]
A = [diff(f, x) diff(f, y); diff(g, x) diff(g, y)]

```
H =

1/2*y^2+1/2*x^2-1/4*x^4



f =

y-q*(x-x^3)



g =

-x+x^3-q*y



ans =

[  0,  0]
[ -1,  0]
[  1,  0]



A =

[ -q*(1-3*x^2),          1]
[     -1+3*x^2,         -q]
```

>> %By looking at the Matrix we can now get a better look at the q on what values it will change the system.
%Now I will plug in the critical points and see how they change by q.

>>x = 0;
y = 0;
CP1 = subs(A)
x = -1;
CP2 = subs(A)
x = 1;
CP3 = subs(A)
syms x y;

```
CP1 =

[ -q,  1]
[ -1, -q]



CP2 =

[ 2*q,   1]
[   2,  -q]



CP3 =

[ 2*q,   1]
[   2,  -q]
```

%Now we must find egien values in terms of q and see how each critical points behaves on different values of q.

>> [V1 D1] = eig(sym(CP1))

```
V1 =

[  1,  1]
[  i, -i]



D1 =

[  i-q,    0]
[    0, -i-q]
```

% examinin this solution q is -, so that means when q > 0 this critical point (0,0) is a CW Spiral sink.

% When q=0 This critical point will behave as a CW center, and when q<0 This point will behave as a CW spiral source.

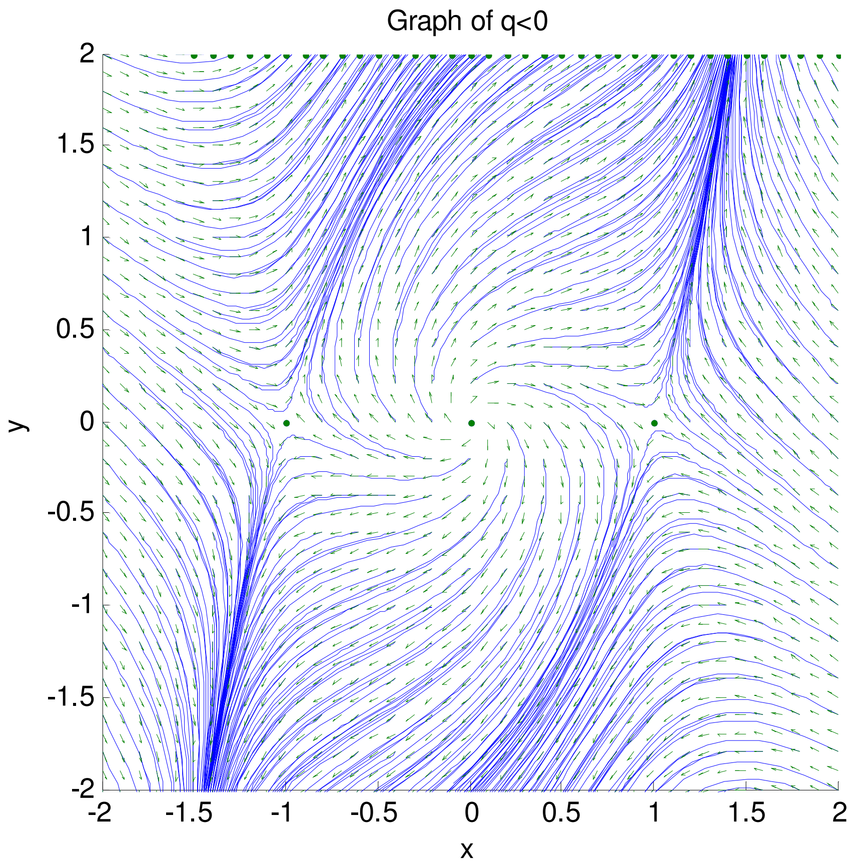
```
>> [V2 D2] = eig(sym(CP2))

V2 =

[ 3/4*q+1/4*(9*q^2+8)^(1/2), 3/4*q-1/4*(9*q^2+8)^(1/2)]
[                         1,                         1]


D2 =

[ 1/2*q+1/2*(9*q^2+8)^(1/2),                         0]
[                         0, 1/2*q-1/2*(9*q^2+8)^(1/2)]

>> solve('1/2*q-1/2*(9*q^2+8)^(1/2)=0')
solve('1/2*q+1/2*(9*q^2+8)^(1/2)=0')

ans =

i

ans =

-sqrt(-1)
```

% From this result we can conclude that the eigen vector will behave the same no matter what value is placed in them. However, examining the egienvalues for point (-1,0) we have a better understanding of what is happening. One of the egienvalues has solution -i which means all the real solutions to that problem will be positive. The second eigen value have a solution i when set to zero, which means no all real values will be negative. Therefore we can conclude this critical point is a saddle independent of the real values placed in it.

```
>> x = 1;
y = 0;
CP3 = subs(A)
[V3 D3] = eig(sym(CP3))

CP3 =

[ 2*q,   1]
[   2,  -q]



V3 =

[                         1,                         1]
```

```
[ -3/2*q+1/2*(9*q^2+8)^(1/2), -3/2*q-1/2*(9*q^2+8)^(1/2)]


D3 =

[ 1/2*q+1/2*(9*q^2+8)^(1/2),                    0]
[                    0, 1/2*q-1/2*(9*q^2+8)^(1/2)]

 >> solve('1/2*q-1/2*(9*q^2+8)^(1/2)=0')
solve('1/2*q+1/2*(9*q^2+8)^(1/2)=0')

ans =

sqrt(-1)


ans =

-sqrt(-1)

%Again this Critical point (1,0) behaves the same way as the (-1,0) behaves. This is because of the solutions that are given by the solve function. For one of the eigenvalues I got i which means that all real values will give negative solution. For the second eigenvalue I got a solution -i which means all real values will give a positive answer. Therefore, we can conclude that this is a saddle point independent of the value of q. The main points which will show different behaviors of this equation are going to be, q<0 q=0 q>0. The only change must be the Spiral source and the spiral sink and the center.
%Graph 1 q<0

>>q = -1;
GP1 =@(t,x)[x(2)-q*(x(1)-x(1)^3);-x(1)+(x(1)^3-q*x(2))];
figure;hold on
for a=-2:.2:2
for b=-2:.2:2
[t,xa]=ode45(GP1,[0 3],[a b]);
plot(xa(:,1),xa(:,2))
end
end
[X,Y]=meshgrid(-2:.4:2,-2:.4:2);
axis([-2 2 -2 2])
hold on
[X, Y] = meshgrid(-2:0.1:2, -2:0.1:2);
U = Y-q*X+q*X.^3;
V = -X+X.^3-q*Y;
L = sqrt(U.^2+V.^2);
quiver(X, Y, U./L, V./L, 0.4)
axis equal tight
xlabel 'x'
```

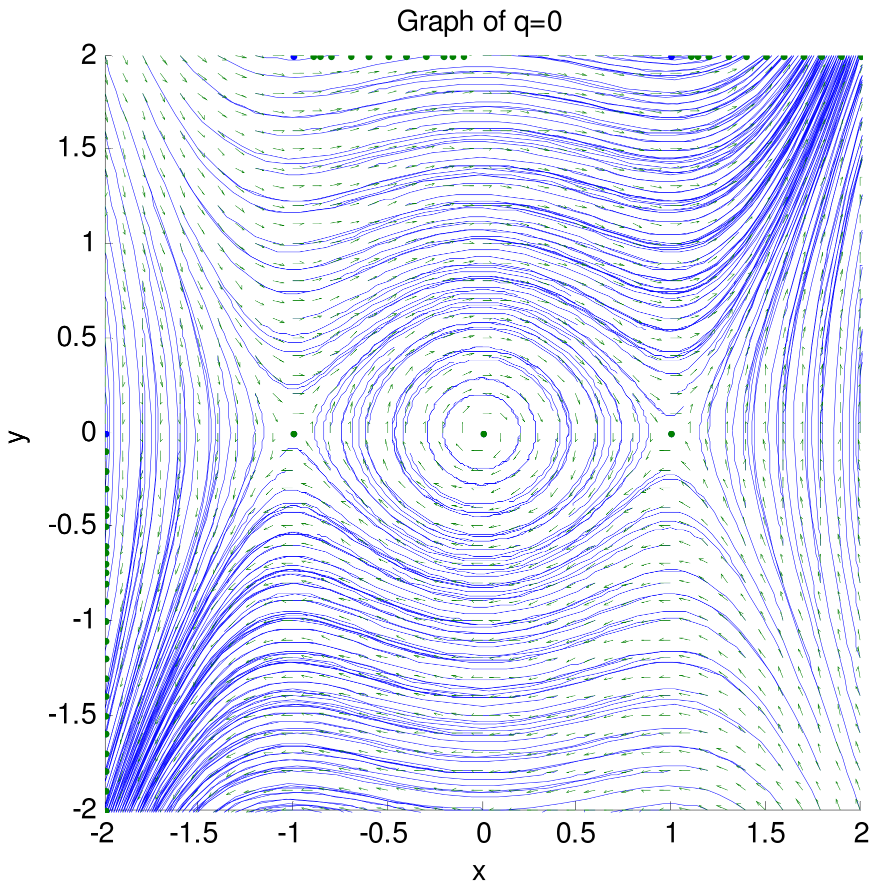
```
ylabel 'y'
title 'Graph of q<0'
axis([-2 2 -2 2])
```



% As it is shown in the graph my prediction was correct for the value of q. The critical point (0,0) is a CW spiral source, and the points (-1,0) and (1,0) are saddle points.
% Graph q=0

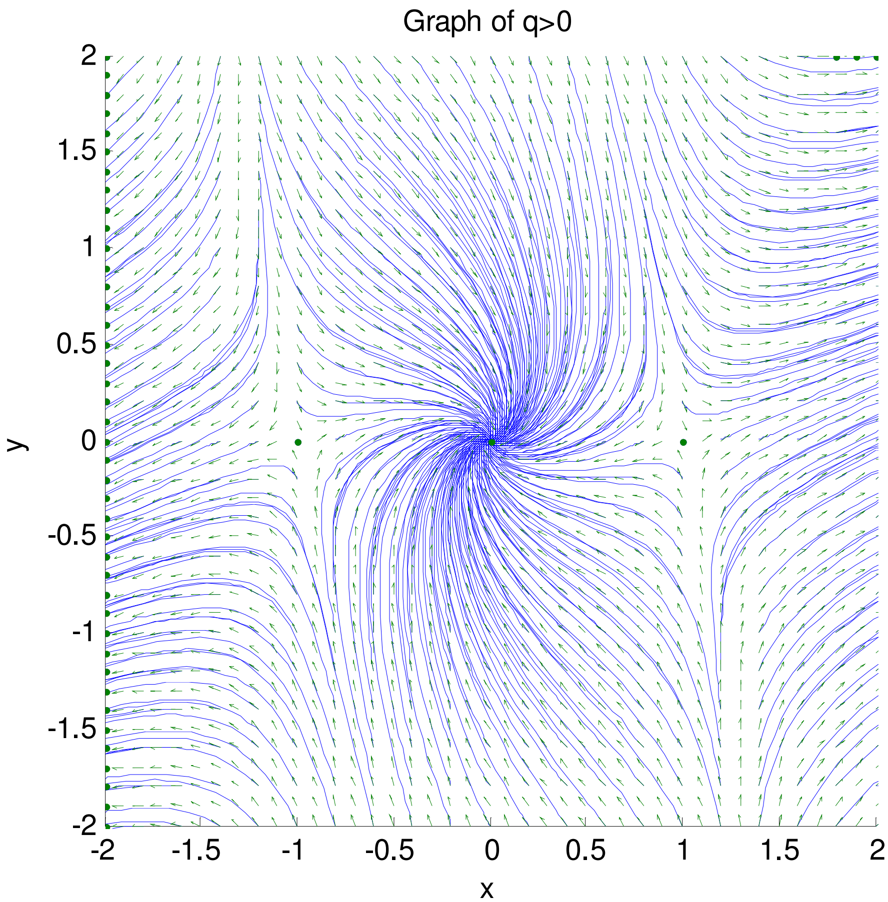
```
q = 0;
GP1 =@(t,x)[x(2)-q*(x(1)-x(1)^3);-x(1)+(x(1)^3-q*x(2))];
figure;hold on
for a=-2:.2:2
for b=-2:.2:2
[t,xa]=ode45(GP1,[0 3],[a b]);
plot(xa(:,1),xa(:,2))
end
end
[X,Y]=meshgrid(-2:.4:2,-2:.4:2);
axis([-2 2 -2 2])
hold on
[X, Y] = meshgrid(-2:0.1:2, -2:0.1:2);
U = Y-q*X+q*X.^3;
V = -X+X.^3-q*Y;
```

```
L = sqrt(U.^2+V.^2);
quiver(X, Y, U./L, V./L, 0.4)
axis equal tight
xlabel 'x'
ylabel 'y'
title 'Graph of q=0'
axis([-2 2 -2 2])
```




```
%The results shown by the graph proves my predictions about the graph. As we can see the critical point
(0,0) is a CW center, and the points (-1,0) and (1,0) are saddle points.
%Graph of q>0
q = 2;
GP1 =@(t,x)[x(2)-q*(x(1)-x(1)^3);-x(1)+(x(1)^3-q*x(2))];
figure;hold on
for a=-2:.2:2
for b=-2:.2:2
[t,xa]=ode45(GP1,[0 3],[a b]);
plot(xa(:,1),xa(:,2))
end
end
[X,Y]=meshgrid(-2:.4:2,-2:.4:2);
```

```
axis([-2 2 -2 2])
hold on
[X, Y] = meshgrid(-2:0.1:2, -2:0.1:2);
U = Y-q*X+q*X.^3;
V = -X+X.^3-q*Y;
L = sqrt(U.^2+V.^2);
quiver(X, Y, U./L, V./L, 0.4)
axis equal tight
xlabel 'x'
ylabel 'y'
title 'Graph of q>0'
axis([-2 2 -2 2])
```



%Similarly my prediction about the third case is also correct. When we examine the graph, when the value q>0 Then the critical point (0,0) is a CW spiral sink and the points (1,0) and (-1,0) are still saddle, the only change in them is the degree of their eigenvectors.

%Therefore we can conclude that our predictions for the values of q by examining each of the critical points individually was correct. We made our predictions based on the eigenvalues of each of the critical points which mapped out an idea of the behavior of the graph based on different values of q.