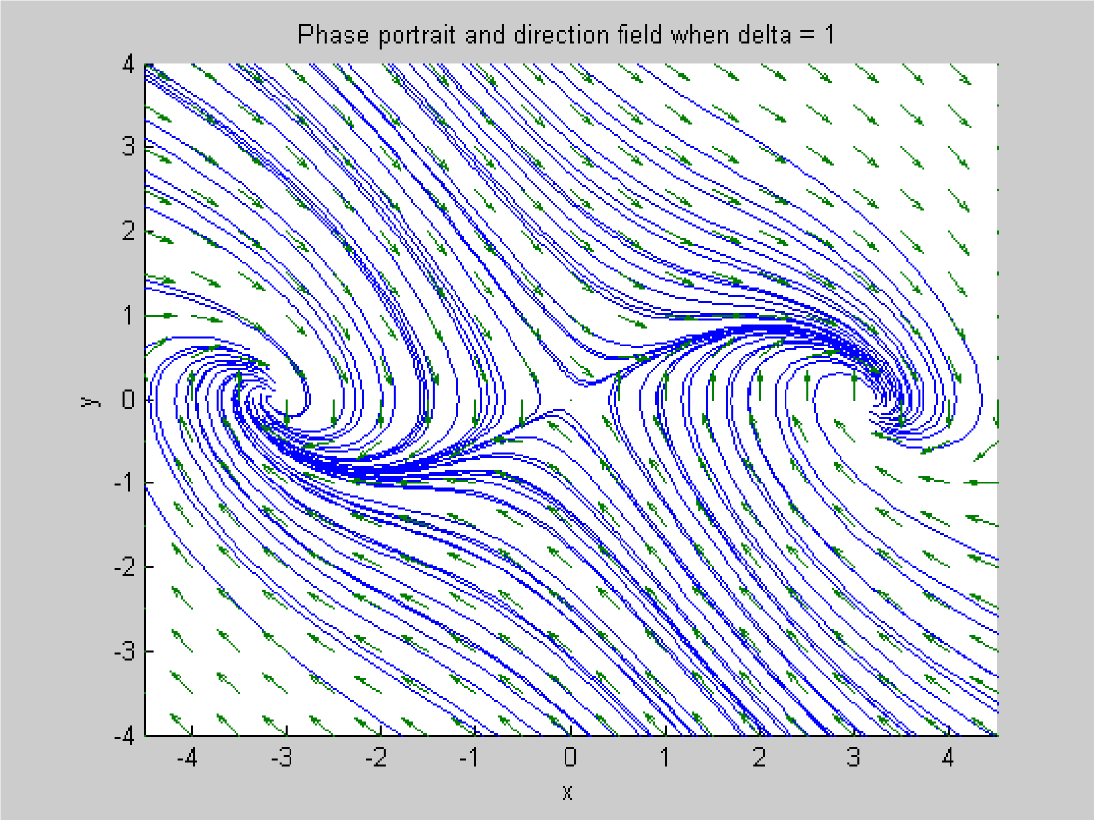
Contents

Extra credit project

```
% Investigation of H(x,y) = (1/2)*y^2 + 1 + cos(x)

% d/dt(x; y) = (Hy; -Hx - (delta)*Hy) as delta moves from 1 to 0
```

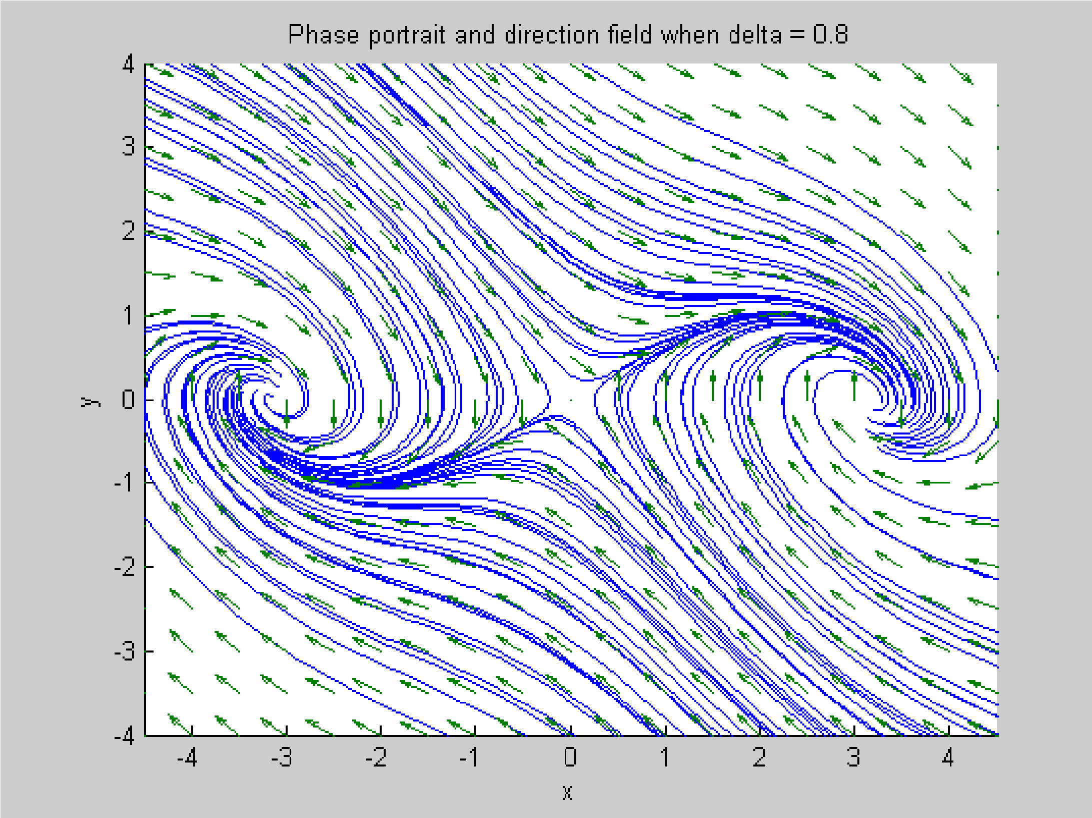
delta = 1

```
f = @(t, x) [x(2); sin(x(1)) - x(2)];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 1'
```



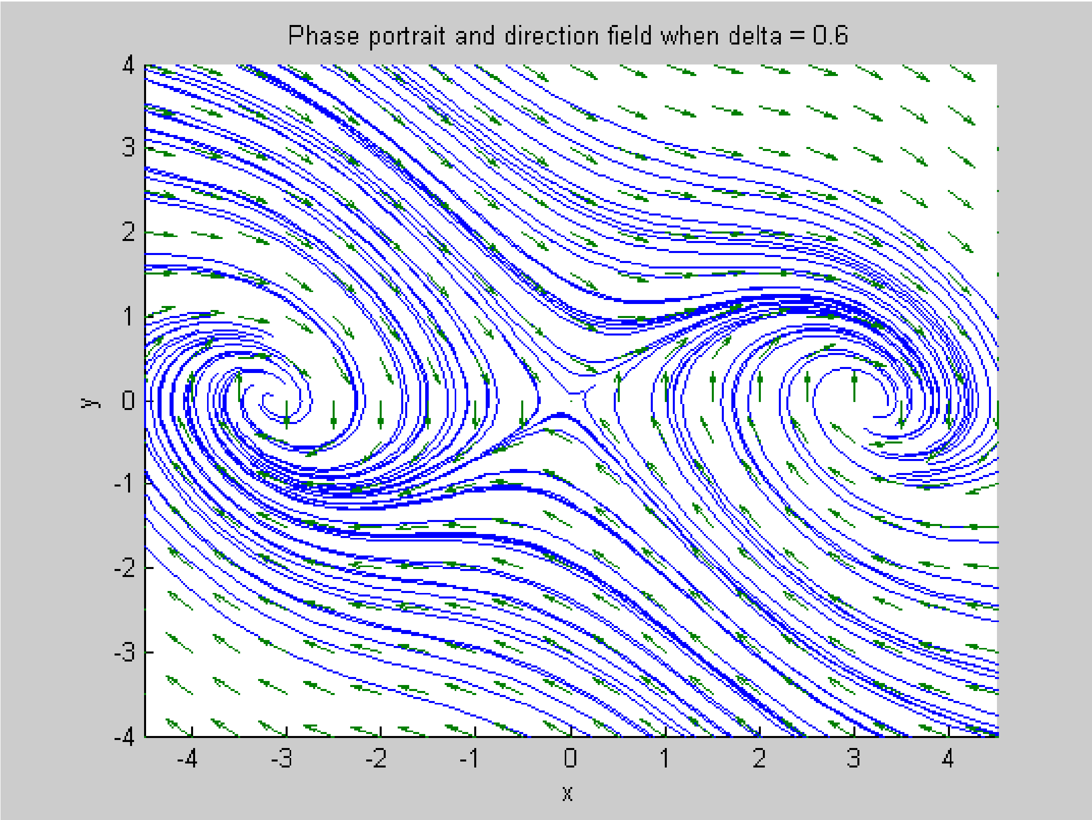
delta = 0.8

```
f = @(t, x) [x(2); sin(x(1)) - .8*x(2)];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - .8*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.8'
```



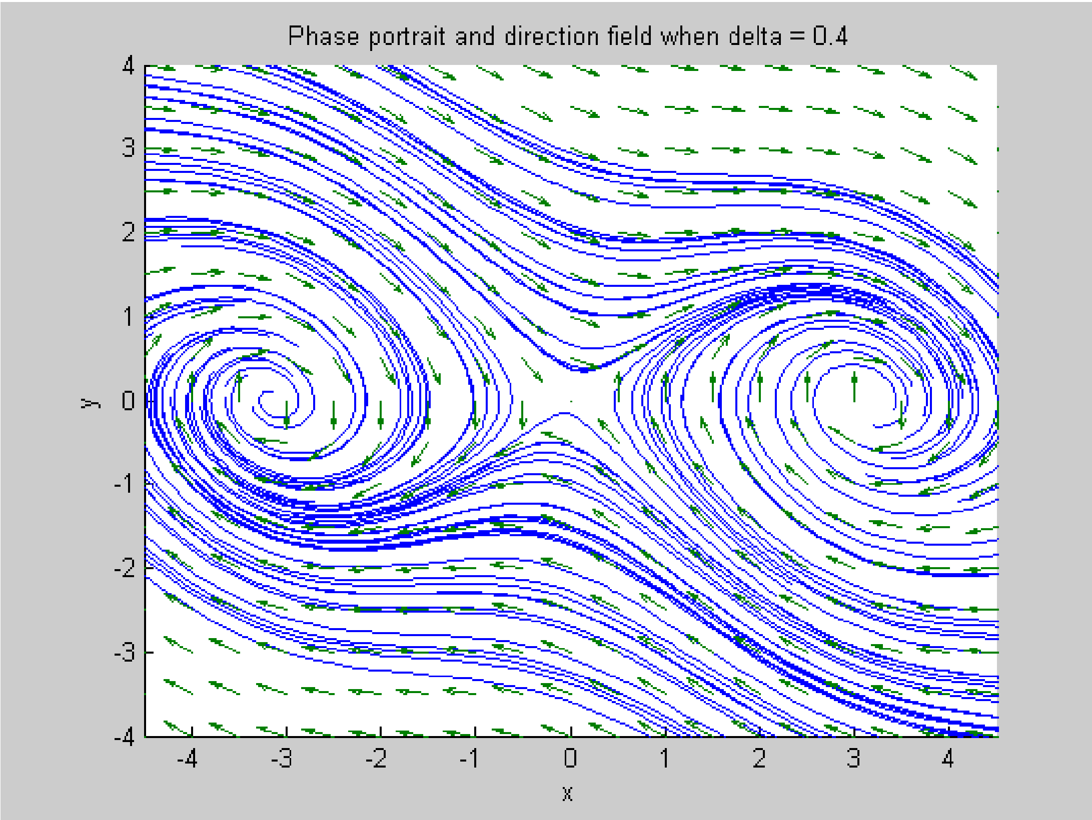
delta = 0.6

```
f = @(t, x) [x(2); sin(x(1)) - .6*x(2)];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - .6*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.6'
```



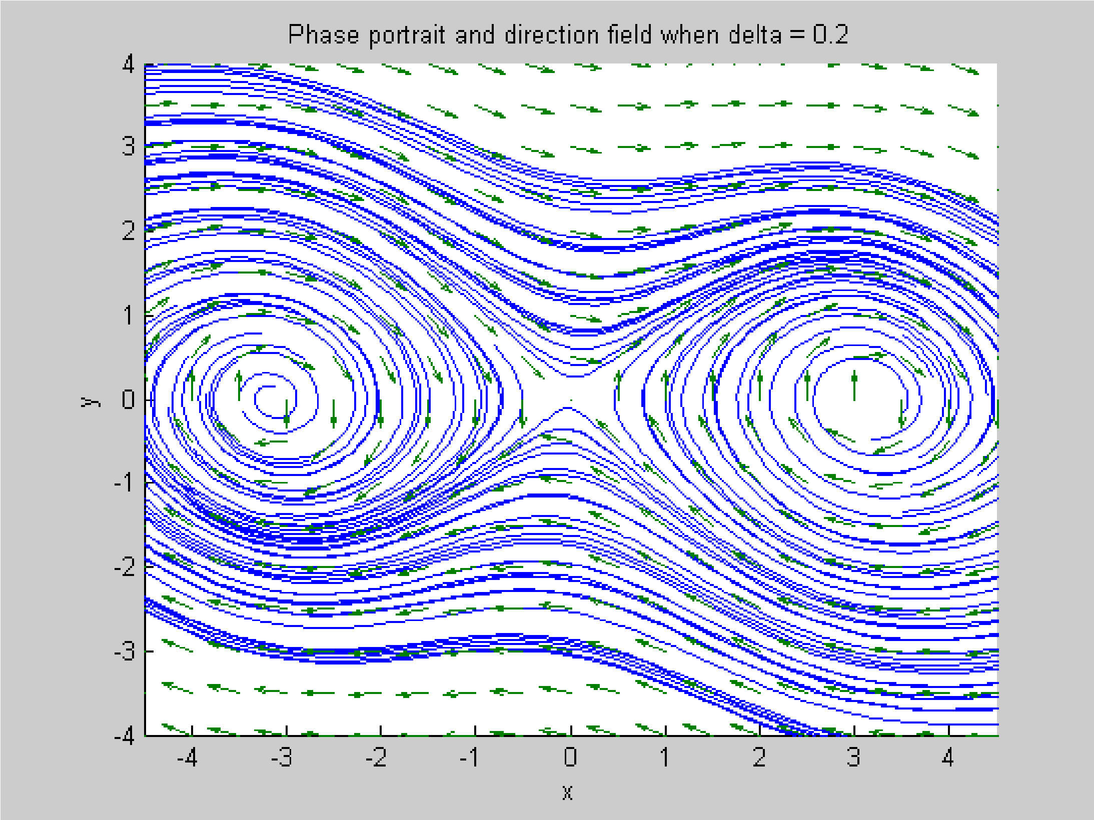
delta = 0.4

```
f = @(t, x) [x(2); sin(x(1)) - .4*x(2)];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - .4*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.4'
```



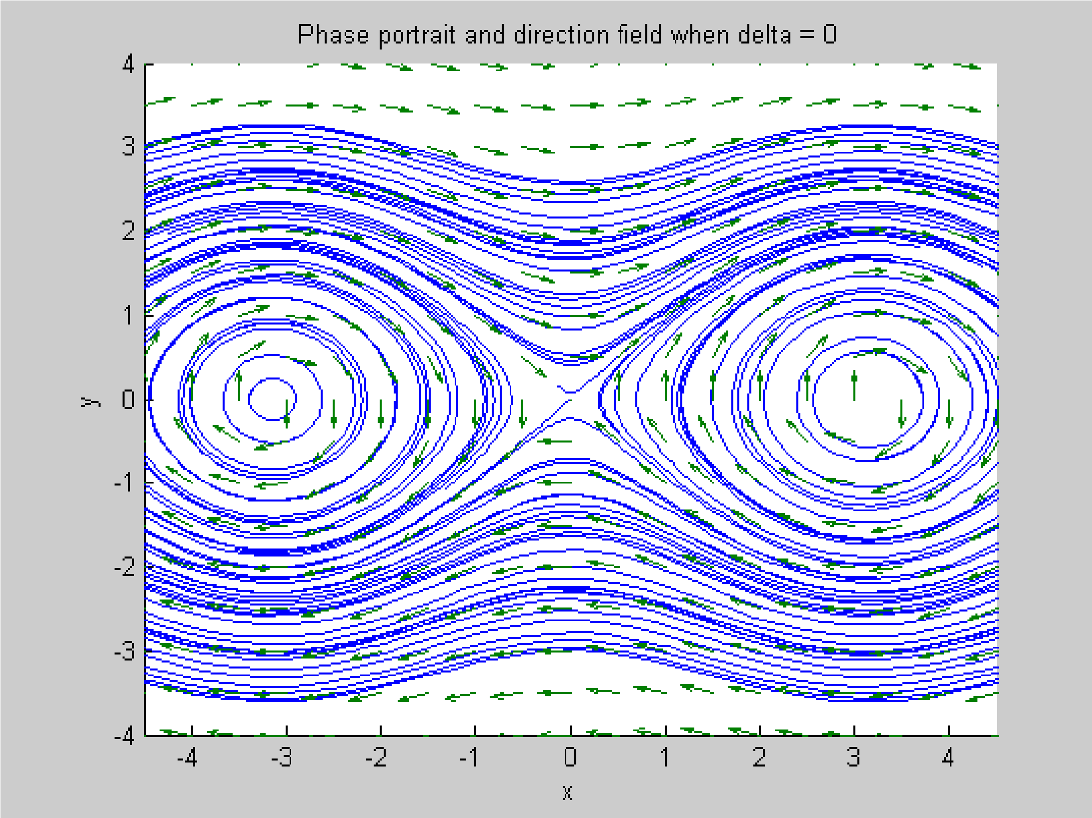
delta = 0.2

```
f = @(t, x) [x(2); sin(x(1)) - .2*x(2)];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X) - .2*Y;
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0.2'
```



delta = 0

```
f = @(t, x) [x(2); sin(x(1))];
figure, hold on
for a = -3:.7:3
    for b = -3:.7:3
        [t, xa] = ode45(f, [0 4], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -4], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ([-4.5 4.5 -4 4])

[X,Y] = meshgrid(-4.5:.5:4.5, -4:.5:4);
U = Y;
V = sin(X);
L = sqrt(U.^2 + V.^2);
quiver(X, Y, U./L, V./L, .5)
xlabel 'x'
ylabel 'y'
title 'Phase portrait and direction field when delta = 0'
```



Observations

```
% There is a very interesting progression in the phase portraits for the
% system as delta goes to 0.  For delta = 1, the origin is an unstable
% saddle point and as the solution curves move away, they are attracted to
% the critical points (-3.14, 0) and (3.14, 0), as they are clockwise
% spiral sinks.

% When delta = 0.8, the saddle point, and spiral sinks remain the same, but
% the sinks are more attracting and there are more solution curves drawn
% in.

% When delta = 0.6, the saddle point begins to appear more horizontal, as
% does the whole phase portrait.  Again, more solution curves are being
% directed about the clockwise spiral sink point.

% When delta = 0.4, the phase portrait is even more horizontal, and the
% slopes far away from the critical points are shifting away from the
% saddle point and more towards the sinks.

% When delta = 0.2, the phase portrait is almost completely horizontal, and
% the spiral sinks are becoming more like centers.

% When delta = 0, the origin is still a saddle point, and the points
% (-3.14, 0) and (3.14, 0) are stable clockwise centers.  The system
% is closed and as a whole moves in the clockwise direction.
```

Published with MATLAB® 7.6