

SERGE MUYA
Math246
Extra-credit

Phase Plane portrait VS contour of function

-MATRIX A

$a_{11} = \mu + 1$, $a_{12} = 1$, $a_{21} = 3$, $a_{22} = \mu - 1$,

-Function: $h(x,y) = 3x^2 - 2xy - y^2$

1) $\mu = -3$

```
ivp='Dx=-2*x+3*y, Dy=x-4*y, x(0)=a, y(0)=b';
```

```
[x, y]=dsolve(ivp, 't');
```

```
xf= @(t, a, b) eval(vectorize(x));
```

```
yf= @(t, a, b) eval(vectorize(y));
```

```
figure; hold on
```

```
t=-3:0.1:3;
```

```
for a=-2:2
```

```
    for b=-2:2
```

```
        plot(xf(t, a, b),yf(t, a, b));
```

```
    end
```

```
end
```

```
axis([-5 5 -5 5])
```

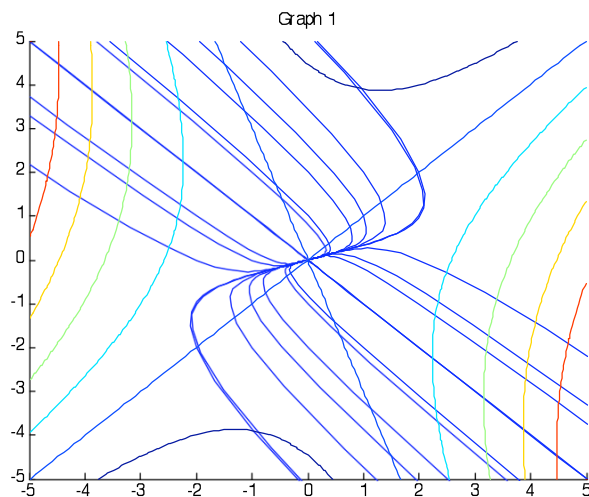
```
[X,Y]=meshgrid(-5:0.1:5, -5:0.1:5);
```

```
hold on
```

```
contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);
```

```
title 'Graph 1'
```

```
hold off
```

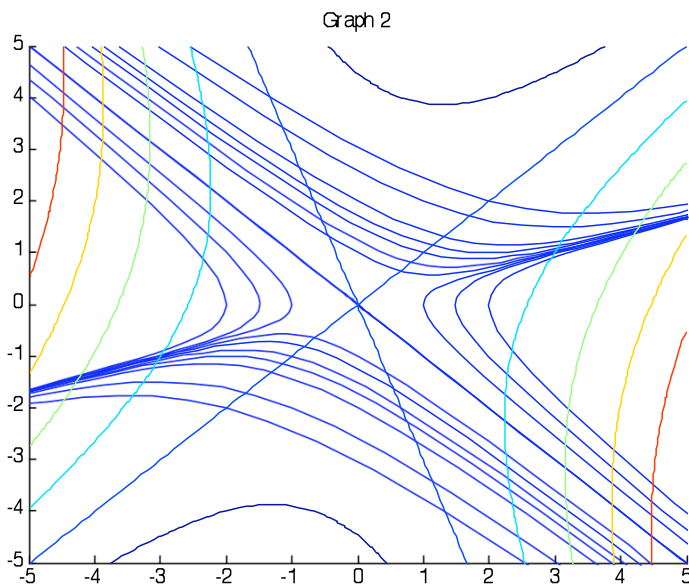


Here we have $\mu = -3$. By changing μ , the matrix A also changes thereby resulting in a unique phase portrait that is plotted in the same graph as the function $h(x, y)$. This superimposition reveals that these two plots are not related. The graph of A is a nodal portrait.

```

2) mu= -1
ivp='Dx=3*y, Dy=x-2*y, x(0)=a, y(0)=b';
[x, y]=dsolve(ivp, 't');
xf= @(t, a, b) eval(vectorize(x));
yf= @(t, a, b) eval(vectorize(y));
figure; hold on
t=-3:0.1:3;
for a=-2:2
    for b=-2:2
        plot(xf(t, a, b),yf(t, a, b));
    end
end
axis([-5 5 -5 5])
[X,Y]=meshgrid(-5:0.1:5, -5:0.1:5);
hold on
contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);
title 'Graph 2'
hold off

```

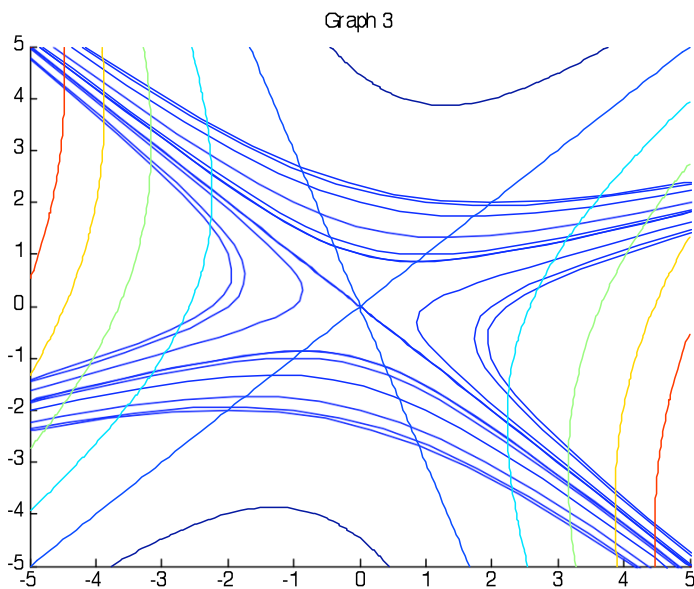


For this superimposition, μ is -1; which resulted in a phase portrait of matrix A that is nodal. The phase portrait and the graph of the function are both nodal.

```

3) mu= 0
ivp='Dx=1*x+3*y, Dy=x-1*y, x(0)=a, y(0)=b';
[x, y]=dsolve(ivp, 't');
xf= @(t, a, b) eval(vectorize(x));
yf= @(t, a, b) eval(vectorize(y));
figure; hold on
t=-3:0.1:3;
for a=-2:2
    for b=-2:2
        plot(xf(t, a, b),yf(t, a, b));
    end
end
hold off
axis([-5 5 -5 5])
[X,Y]=meshgrid(-5:0.1:5, -5:0.1:5);
hold on
contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);
title 'Graph 3'
hold off

```

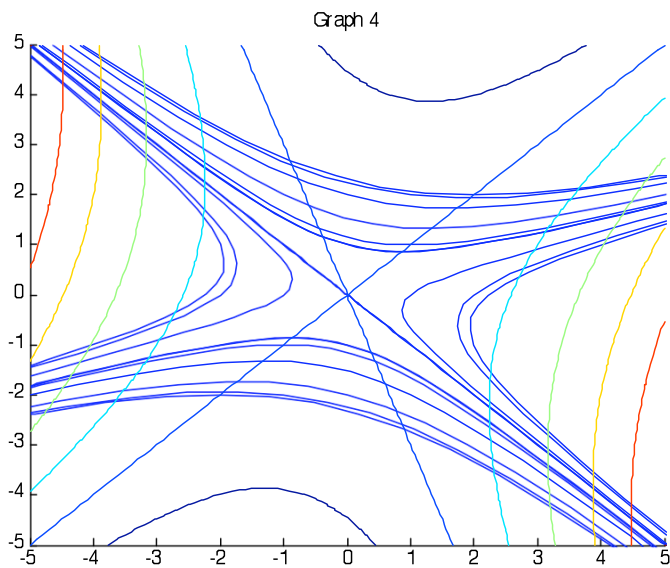


Here mu is 0. With mu=0, the phase portrait of A and the graph of the function have closely looking graphs.

```

4) mu= 2
ivp='Dx=1*x+3*y, Dy=x-1*y, x(0)=a, y(0)=b';
[x, y]=dsolve(ivp, 't');
xf= @(t, a, b) eval(vectorize(x));
yf= @(t, a, b) eval(vectorize(y));
figure; hold on
t=-3:0.1:3;
for a=-2:2
    for b=-2:2
        plot(xf(t, a, b),yf(t, a, b));
    end
end
hold off
axis([-5 5 -5 5])
[X, Y]=meshgrid(-5:0.1:5, -5:0.1:5);
hold on
contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);
title 'Graph 4'
hold off

```

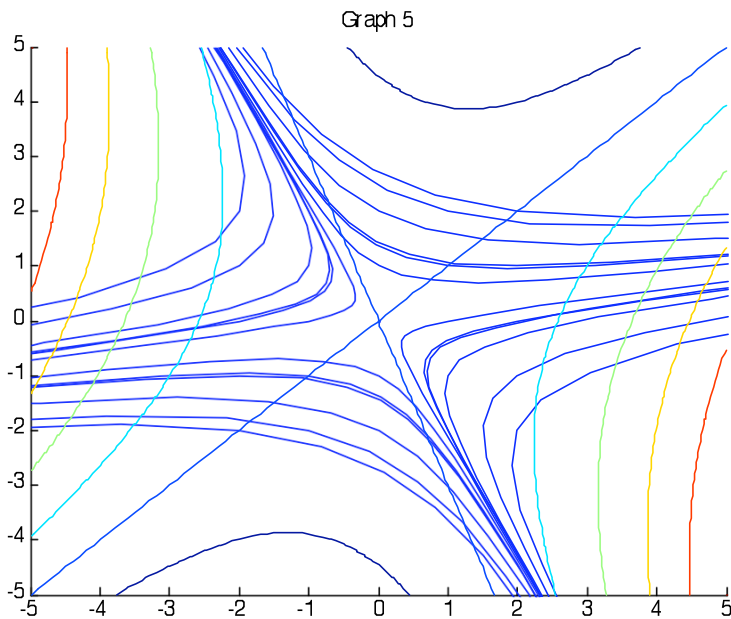


Here $\mu=2$. The graphs of the matrix A and of the function $h(x, y)$ are getting more closely aligned with each others.

```

5) mu= 3
ivp='Dx=4*x+3*y, Dy=x-2*y, x(0)=a, y(0)=b';
[x, y]=dsolve(ivp, 't');
xf= @(t, a, b) eval(vectorize(x));
yf= @(t, a, b) eval(vectorize(y));
figure; hold on
t=-3:0.1:3;
for a=-2:2
    for b=-2:2
        plot(xf(t, a, b),yf(t, a, b));
    end
end
hold off
axis([-5 5 -5 5])
[X,Y]=meshgrid(-5:0.1:5, -5:0.1:5);
hold on
contour(X, Y, 3.*X.^2 - 2.*X.*Y- Y.^2);
title 'Graph 5'
hold off

```



This is the last change of μ that also changes matrix A . As we can see in the phase portrait and the graph have closely similar graphs. Although, this is not perfect, the two graphs are shown to be related in some way.