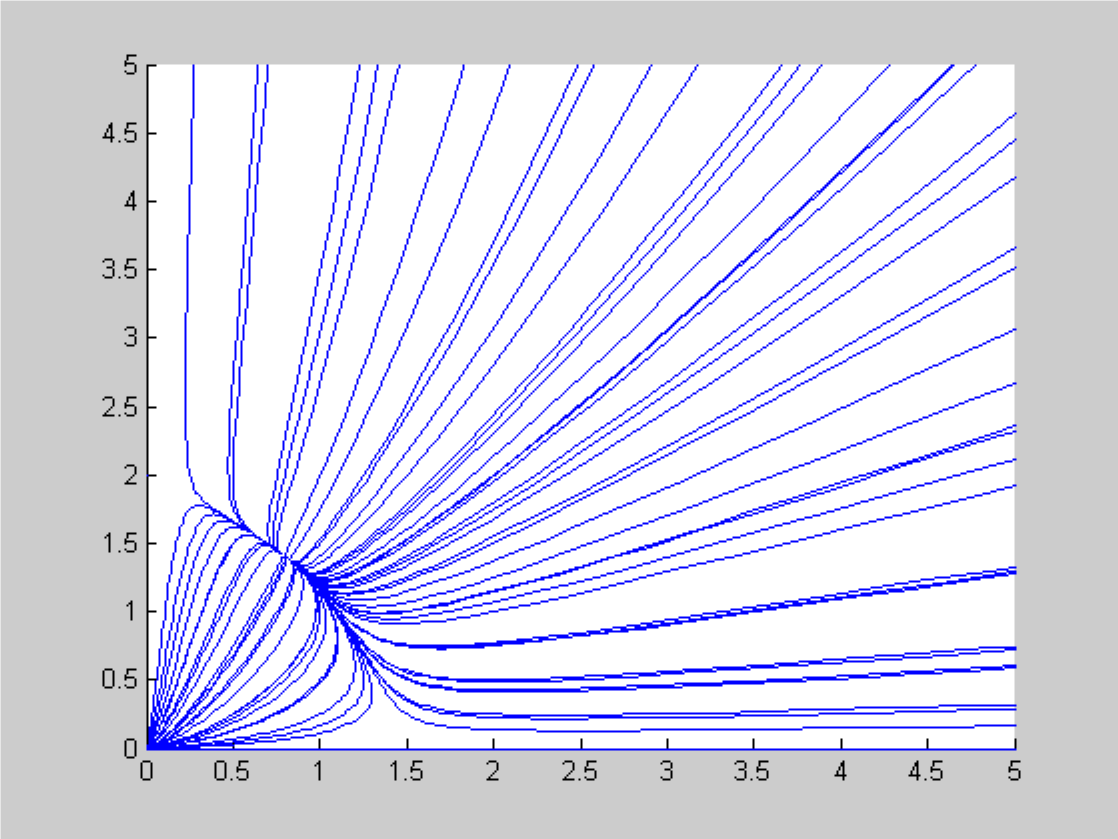
Ben Nguyen Math246 Sec0232 Cynthia Bossard May 12, 2009

Phase Portrait w/ Alpha=0, Critical Points

```
A=solve('x*(3/2-x-.5*y)=0', 'y*(2-(1-0/2)*y-(3/4+(3/4)*0)*x)=0')
A.x
A.y

figure; hold on
syms alpha
alpha = 0;
f=@(t, x) [x(1)*(1.5-x(1)-.5*x(2)) ; x(2)*(2-(1-alpha/2)*x(2)-
(3/4+(3/4)*alpha)*x(1))];
for a = -2:.25:2
    for b = -2:.25:2
        [t, xa] = ode45(f, [0 3], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -3], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ( [0 5 0 5])
A =

    x: [4x1 sym]
    y: [4x1 sym]


ans =

   0
   0
 1.5
 0.8


ans =

   0
 2.0
   0
 1.4
```



Critical Point Anaylsis

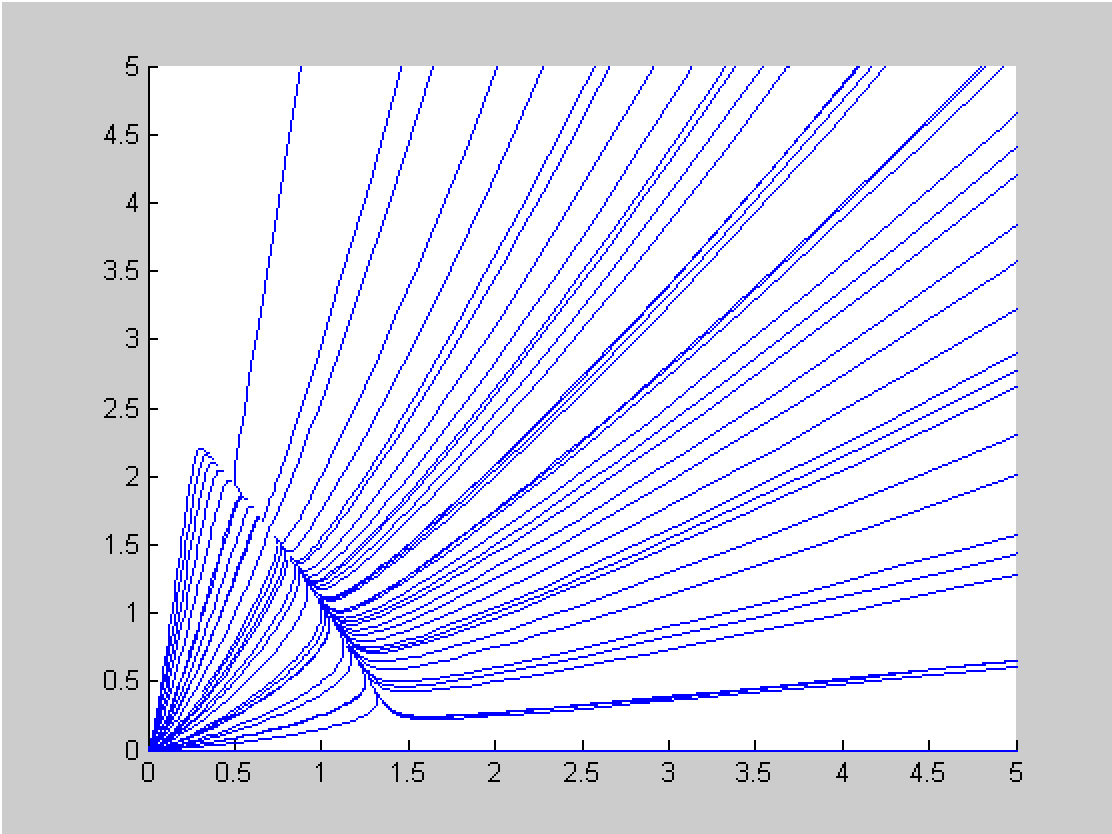
```
x=0
y=0
C=[3/2-y/2-2*x -x/2; 2-2*y-(3*x)/4 2-2*y-(3*x)/4]
[xi, R1]=eig(C)
% eigenvalues = 2, 1.5 therefore (0,0) is nodal source
clear all
x=0
y=2
C2=[3/2-y/2-2*x -x/2; 2-2*y-(3*x)/4 2-2*y-(3*x)/4]
[xi, R2]=eig(C2)
% eigenvalues = -2, .5 therefore (0,2) is saddle
clear all
x=1.5
y=0
C3=[3/2-y/2-2*x -x/2; 2-2*y-(3*x)/4 2-2*y-(3*x)/4]
[xi, R3]=eig(C3)
%eigenvalues = -1.18, .55 therefore (1.5,0) is saddle
clear all
x=.8
y=1.4
C4=[3/2-y/2-2*x -x/2; 2-2*y-(3*x)/4 2-2*y-(3*x)/4]
[xi, R4]=eig(C4)
%eigenvalues = -.293, -1.9 therefore (.8,1.4) is nodal sink
x =
```

```
     0


y =

     0


C =

    1.5000         0
    2.0000    2.0000


xi =

         0    0.2425
    1.0000   -0.9701


R1 =

    2.0000         0
         0    1.5000


x =

     0


y =

     2


C2 =

    0.5000         0
   -2.0000   -2.0000


xi =

         0    0.7809
    1.0000   -0.6247


R2 =

   -2.0000         0
         0    0.5000


x =

    1.5000
```

```
y =

     0


C3 =

   -1.5000   -0.7500
    0.8750    0.8750


xi =

   -0.9201    0.3427
    0.3916   -0.9394


R3 =

   -1.1808         0
         0    0.5558


x =

    0.8000


y =

    1.4000


C4 =

   -0.8000   -0.4000
   -1.4000   -1.4000


xi =

    0.6200    0.3400
   -0.7846    0.9404


R4 =

   -0.2938         0
         0   -1.9062
```

Phase Portrait w/ Alpha=.5, Critical Points

```
A=solve('x*(3/2-x-.5*y)=0', 'y*(2-(1-.5/2)*y-(3/4+(3/4)*.5)*x)=0')
A.x
A.y
figure; hold on
syms alpha
alpha = .5;
f=@(t, x) [x(1)*(1.5-x(1)-.5*x(2)) ; x(2)*(2-(1-alpha/2)*x(2)-
(3/4+(3/4)*alpha)*x(1))];
for a = -2:.25:2
    for b = -2:.25:2
        [t, xa] = ode45(f, [0 3], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -3], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ( [0 5 0 5])
A =

    x: [4x1 sym]
    y: [4x1 sym]


ans =

                                  0
                                  0
                                1.5
 0.66666666666666666666666666666667


ans =

                                 0
 2.6666666666666666666666666666667
                                 0
 1.6666666666666666666666666666667
```



Critical Point Anaylsis

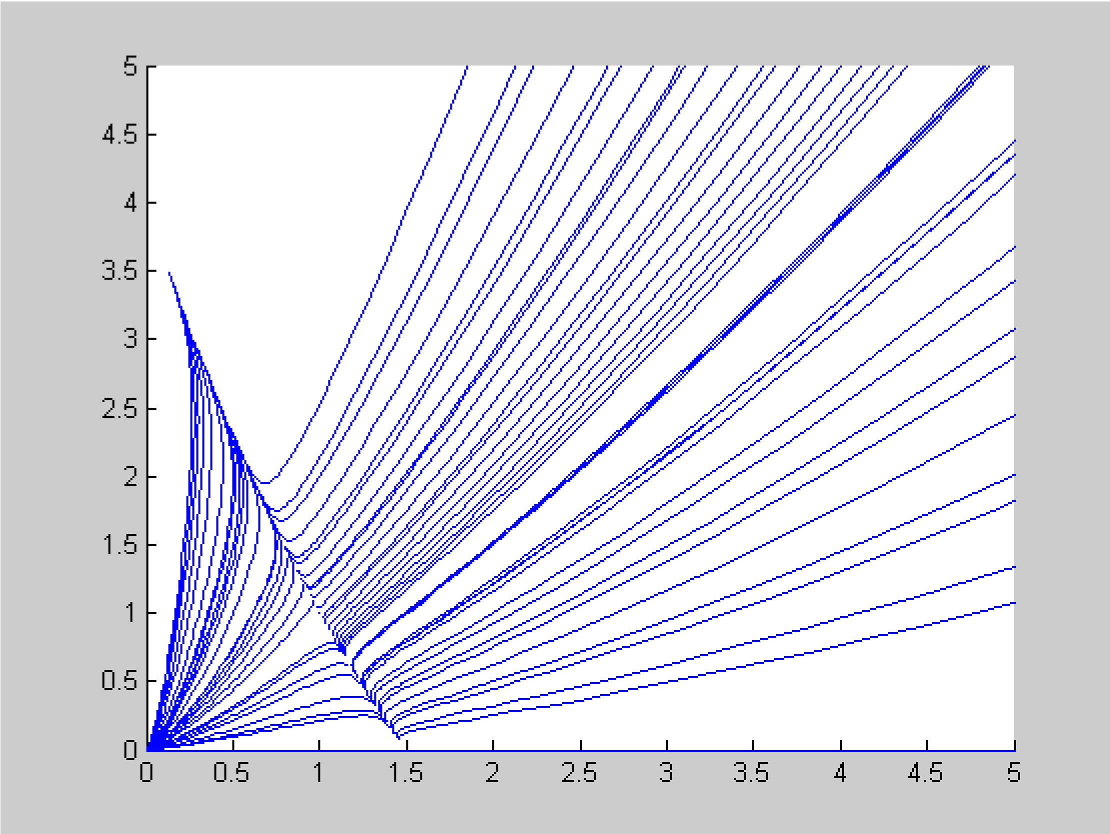
```
clear all
x=0
y=0
C=[3/2-y/2-2*x -x/2; 2-(3*y)/2-(9*x)/8 2-(3*y)/2-(9*x)/8]
[xi, R1]=eig(C)
% eigenvalues = 2, 1.5 therefore (0,0) is nodal source
clear all
x=0
y=8/3
C2=[3/2-y/2-2*x -x/2; 2-(3*y)/2-(9*x)/8 2-(3*y)/2-(9*x)/8]
[xi, R2]=eig(C2)
% eigenvalues = -2, .1667 therefore (0,8/3) is saddle
clear all
x=1.5
y=0
C3=[3/2-y/2-2*x -x/2; 2-(3*y)/2-(9*x)/8 2-(3*y)/2-(9*x)/8]
[xi, R3]=eig(C3)
% eigenvalues = -1.35, 1.172 therefore (1.5,0) is saddle
clear all
x=2/3
y=5/3
C4=[3/2-y/2-2*x -x/2; 2-(3*y)/2-(9*x)/8 2-(3*y)/2-(9*x)/8]
[xi, R4]=eig(C4)
% eigenvalues = -2.5,-1.667 therefore (2/3, 5/3) is nodal sink
x =
```

```
     0


y =

     0


C =

    1.5000         0
    2.0000    2.0000


xi =

         0    0.2425
    1.0000   -0.9701


R1 =

    2.0000         0
         0    1.5000


x =

     0


y =

    2.6667


C2 =

    0.1667         0
   -2.0000   -2.0000


xi =

         0    0.7348
    1.0000   -0.6783


R2 =

   -2.0000         0
         0    0.1667


x =
```

```
    1.5000


y =

     0


C3 =

   -1.5000   -0.7500
    0.3125    0.3125


xi =

   -0.9830    0.4092
    0.1837   -0.9124


R3 =

   -1.3599         0
         0    0.1724


x =

    0.6667


y =

    1.6667


C4 =

   -0.6667   -0.3333
   -1.2500   -1.2500


xi =

    0.6247    0.3162
   -0.7809    0.9487


R4 =

   -0.2500         0
         0   -1.6667
```

Phase Portrait w/ Alpha=1, Critical Points

```
A=solve('x*(3/2-x-.5*y)=0', 'y*(2-(1-1/2)*y-(3/4+(3/4)*1)*x)=0')
A.x
A.y

figure; hold on
syms alpha
alpha = 1;
f=@(t, x) [x(1)*(1.5-x(1)-.5*x(2)) ; x(2)*(2-(1-alpha/2)*x(2)-
(3/4+(3/4)*alpha)*x(1))];
for a = -2:.25:2
    for b = -2:.25:2
        [t, xa] = ode45(f, [0 3], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -3], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
axis ( [0 5 0 5])
A =

    x: [4x1 sym]
    y: [4x1 sym]


ans =

   0
   0
 1.5
 1.0


ans =

   0
 4.0
   0
 1.0
```



Critical Point Anaylsis

```
x=0
y=0
C=[3/2-y/2-2*x -x/2 ; 2-y-(3*x)/2 2-y-(3*x)/2]
[xi, R1]=eig(C)
% eigenvalues = 2, 1.5 therefore (0,0) is nodal source
clear all
x=0
y=4
C2=[3/2-y/2-2*x -x/2 ; 2-y-(3*x)/2 2-y-(3*x)/2]
[xi, R2]=eig(C2)
% eigenvalues = -2, -5 therefore (0,4) is nodal sink
clear all
x=1.5
y=0
C3=[3/2-y/2-2*x -x/2 ; 2-y-(3*x)/2 2-y-(3*x)/2]
[xi, R3]=eig(C3)
% eigenvalues = -1.63, -.1147 therefore (1.5,0) is nodal sink
clear all
x=1
y=1
C4=[3/2-y/2-2*x -x/2 ; 2-y-(3*x)/2 2-y-(3*x)/2]
[xi, R4]=eig(C4)
% eigenvalues = -1.3, -.19 therefore (1,1) is nodal sink
x =
```

```
     0


y =

     0


C =

    1.5000         0
    2.0000    2.0000


xi =

         0    0.2425
    1.0000   -0.9701


R1 =

    2.0000         0
         0    1.5000


x =

     0


y =

     4


C2 =

   -0.5000         0
   -2.0000   -2.0000


xi =

         0    0.6000
    1.0000   -0.8000


R2 =

   -2.0000         0
         0   -0.5000


x =

    1.5000
```

```
y =

     0


C3 =

   -1.5000   -0.7500
   -0.2500   -0.2500


xi =

   -0.9841    0.4761
   -0.1776   -0.8794


R3 =

   -1.6353         0
         0   -0.1147


x =

     1


y =

     1


C4 =

   -1.0000   -0.5000
   -0.5000   -0.5000


xi =

   -0.8507    0.5257
   -0.5257   -0.8507


R4 =

   -1.3090         0
         0   -0.1910
```

CONCLUSION

Dx/dt=x(1.5-x-.5y)   Dy/dt=y(2-(1-alpha/2)y-(3/4+(3/4)alpha)x)

In this project I am evaluating the properties of changing alpha in the solutions of the system of competing species. In this project I examined three values for alpha; 0, .5 and 1. For each value of alpha, I found the critical points, plotted the phase portraits, found the eigen pairs and determined the trajectory properties at each of the critical points.

In finding the critical points for the values of alpha, I noticed each set of critical points had the points (0,0) and (1.5,0). This has to do with the dx/dt equation staying constant. Also each set of critical points had points (0,y) and (x,y). For the point (0,y) as alpha increased so did. At alpha = 0, y=2, at alpha=.5, y=8/3 and at alpha=1, y=4. For the point of (x,y) for each respective alpha value, x and y were always positive.

The trajectory at the critical point (0,0), were all nodal sources and all had the eigen values 2 and 1.5. Therefore alpha did not effect this critical point. For the point of (0,y) where y increased as alpha increased, all the alphas had an eigen value of -2, but for the other eigen value; as alpha increased, the eigen value decreased and eventually became negative. At alpha=0, point (0,2) had the eigen values of -2 and 5 therefore is a saddle point. At alpha=.5, point (0,8/3) had the eigen values of -2 and .1667, therefore is also a saddle. At alpha=1, point(0,4) had the egien values -2 and -5, therefore is a nodal sink. For the critical point (1.5, 0) of the different alpha values, did not have a noticeable trend in the eigenvalues, but as alpha increases, the trajectory at this critical point becomes more and more attractive. For alpha=0, the critical point is a saddle. For alpha =.5 , the critical point is still a saddle, but when alpha =1 the critical point is a nodal sink. For the point (x,y) of each respective alpha, the trajectory were all nodal sinks. Therefore alpha did not change the trajectory type for the critical point.

In summary the critical point (0,0) for all the values of alpha, had the same trajectory. Alpha did not effect this critical point. For the point (0,y) as alpha increased, so did y and one of the eigenvalues became more and more negative, while for all the different values of alpha had an eigenvalue of -2. The critical point (1.5, 0) for all the values of alpha, as alpha increased, the trajectory became more and more attractive. This is also the same for the respective critical points (0,y) for the range of alphas. Finally the point (x, y) for each respective critical point all had a nodal sink trajectory.