

Extra credit

b) For the matrix $A =$

$$m+1 \quad -13m-1$$

% with $m \in [-2; 2]$

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% We have:  $(\Delta) = (m)^2 - \det(A)$ 
            $= (m)^2 - [(m)^2 - 2]$ 
            $= 2$  or  $(\Delta)^{1/2} = 2^{1/2}$ 

% Since  $(\Delta) > 0$ , we can conclude that there are 2 simple real roots. And
% thus, the plot of it can either be: Nodal Sink if  $m < -2^{1/2}$ ; Nodal
% source if  $m > 2^{1/2}$ ; saddle if  $-2^{1/2} < m < 2^{1/2}$ ; Linear Sink if  $m =$ 
%  $-2^{1/2}$ , Linear source if  $m = 2^{1/2}$ 

% In order to illustrate these predictions, particular values of  $m$  is chosen
% that can satisfy the conditions of 5 cases above

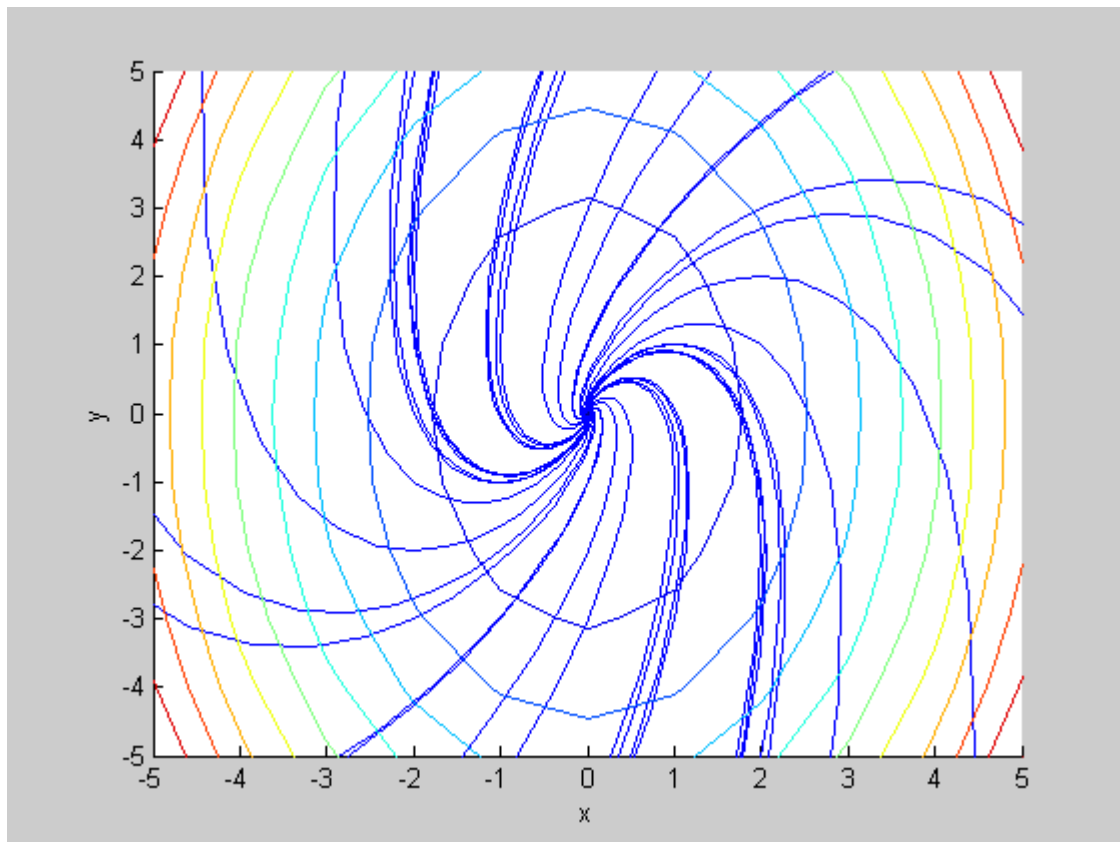
% and also staying in the interval  $[-2; 2]$ 

%-----
% A>  $m = -2$ 
ivp = 'Dx = -1*x - y, Dy = 3*x - 3*y, x(0) = a, y(0) = b';
[x, y] = dsolve(ivp, 't');
xf = @(t, a, b) eval(vectorize(x));
yf = @(t, a, b) eval(vectorize(y));
figure; hold on
t = -3:0.1:3;
for a = -2:2
    for b = -3:3
        plot(xf(t, a, b), yf(t, a, b))
    end
end
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end
[X,Y]=meshgrid(-5:5,-5:5);
contour(X, Y, 3*X.^2 - 2*X*Y + Y.^2)
hold off
axis ([-5 5 -5 5])
xlabel 'x'
ylabel 'y'
% the plot is stable Nodal sink (attractive) lies inside the contour of the
% function: 3 x^2 - 2 x y + y^2

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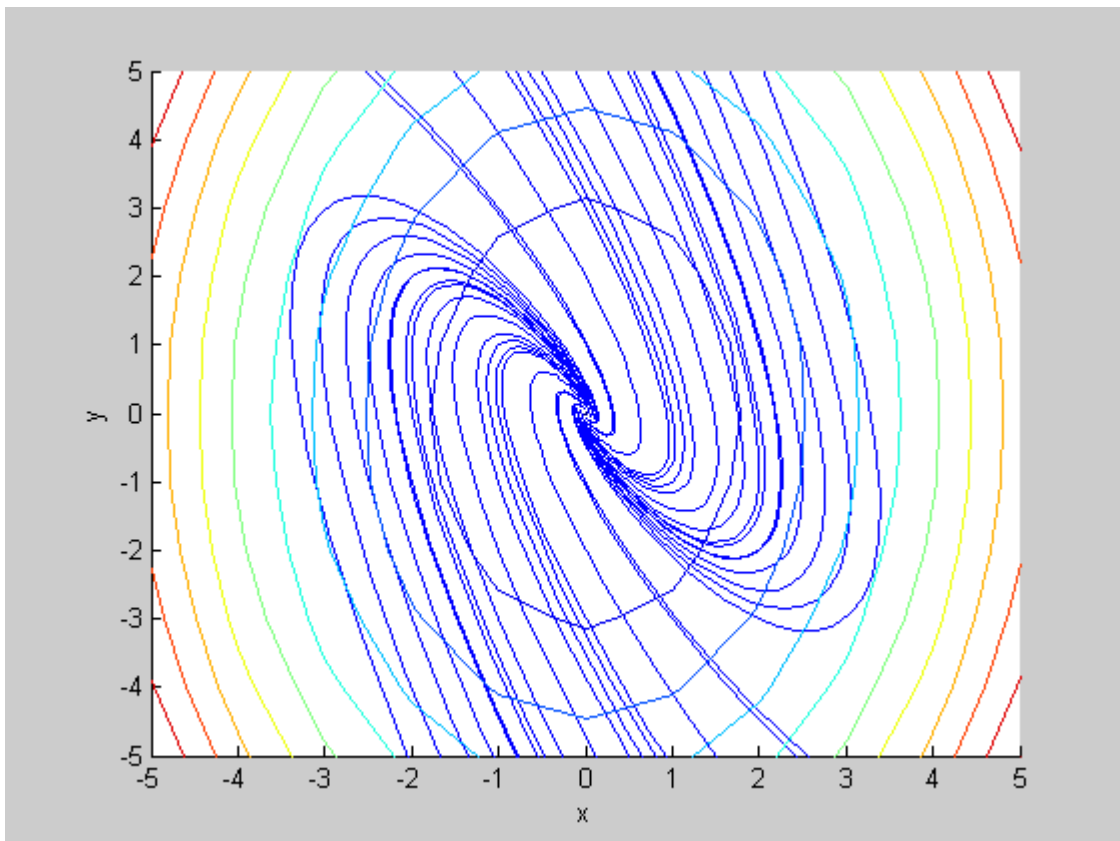
% B> m= -2^(1/2)
ivp='Dx= (-2^(1/2)+1)*x-y,Dy=3*x-(-2^(1/2)-1)*y,x(0)=a,y(0)=b';
[x,y]=dsolve(ivp,'t');
xf=@(t,a,b)eval(vectorize(x));
yf=@(t,a,b)eval(vectorize(y));
figure;hold on

```

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t= -3:0.1:3;
for a= -2:2
    for b=-3:3
        plot(xf(t,a,b),yf(t,a,b))
    end
end
[X,Y]=meshgrid(-5:5,-5:5);
contour(X, Y, 3*X.^2 - 2*X*Y + Y.^2)
hold off
axis ([-5 5 -5 5])
xlabel 'x'
ylabel 'y'
% the matrix id Linear sink (stable) lies inside the contour of the
% function: 3 x^2 - 2 x y + y^2

```

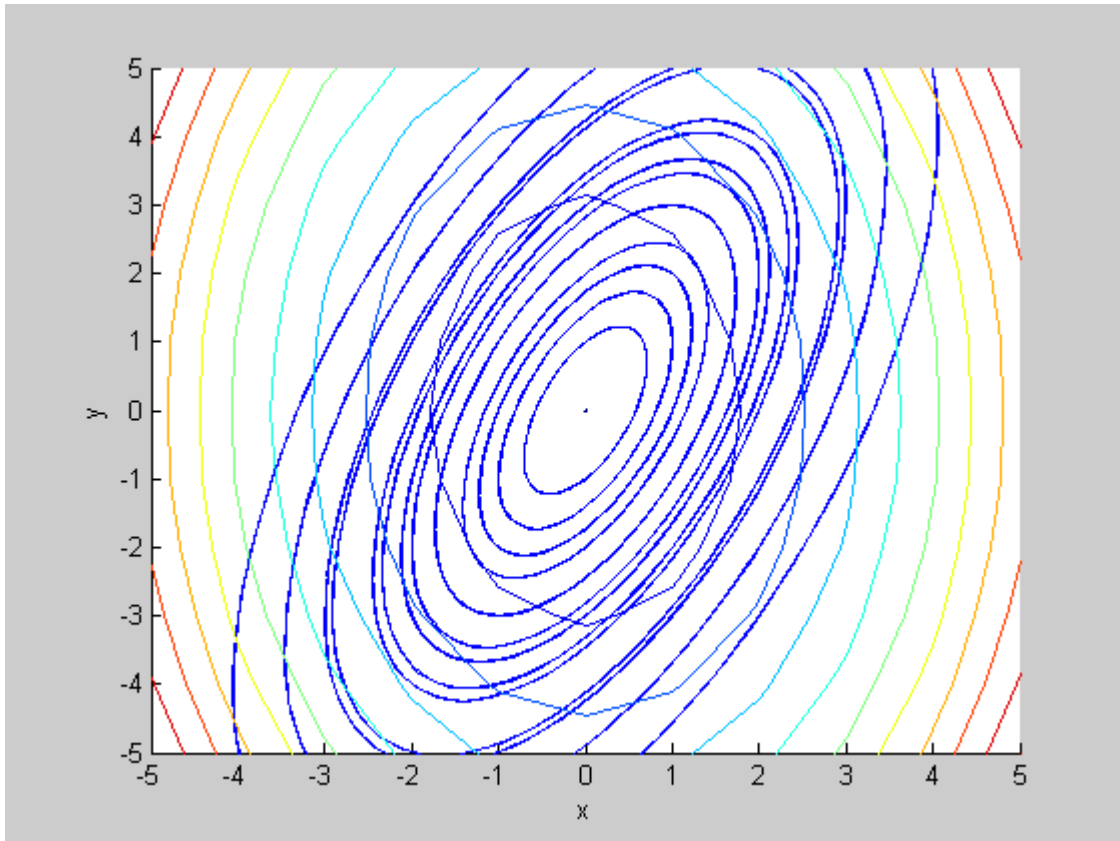


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%C> m=0
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ivp='Dx= x-y,Dy=3*x-y,x(0)=a,y(0)=b';
[x,y]=dsolve(ivp,'t');
xf=@(t,a,b)eval(vectorize(x));
yf=@(t,a,b)eval(vectorize(y));
figure;hold on
t= -3:0.1:3;
for a= -2:2
    for b=-3:3
        plot(xf(t,a,b),yf(t,a,b))
    end
end
[X,Y]=meshgrid(-5:5,-5:5);
contour(X, Y, 3*X.^2 - 2*X*Y + Y.^2)
hold off
axis ([-5 5 -5 5])
xlabel 'x'
ylabel 'y'
% the matrix id Saddle (unstable) lies inside the contour of the
% function: 3 x^2 - 2 x y + y^2

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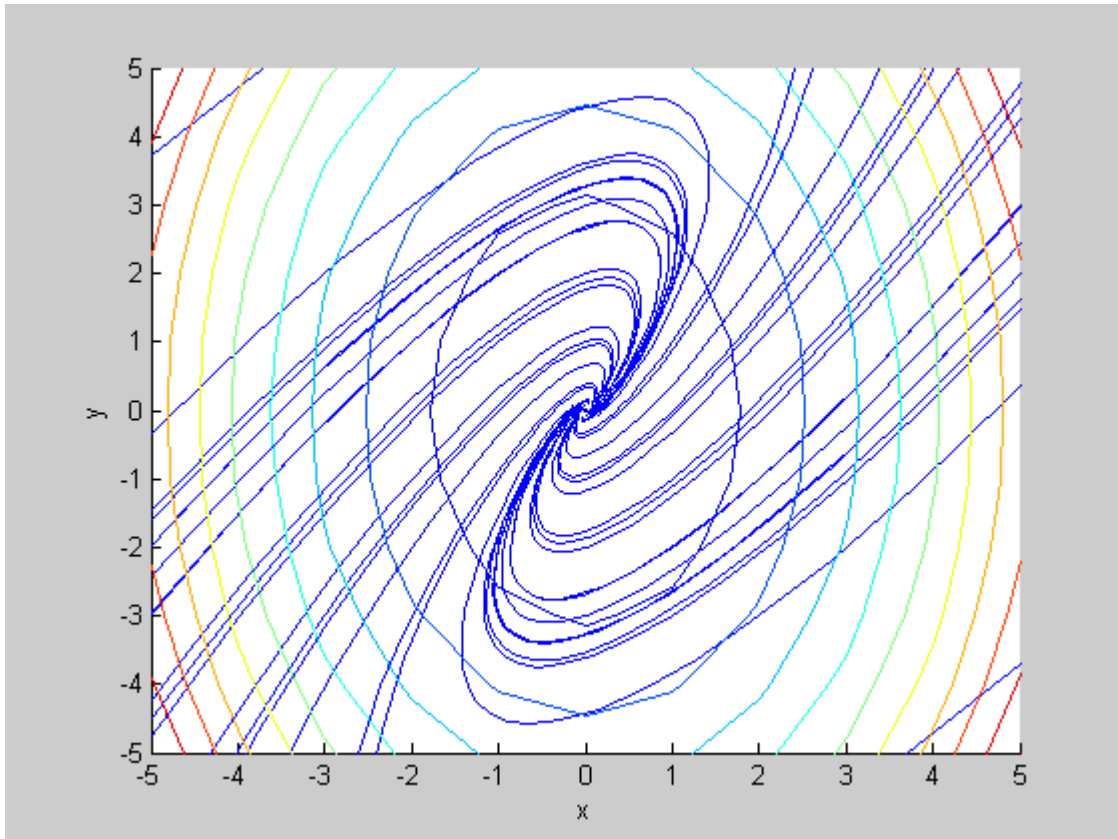
%D> m= 2^(1/2)
ivp='Dx= (2^(1/2)+1)*x-y,Dy=3*x-(2^(1/2)-1)*y,x(0)=a,y(0)=b';
[x,y]=dsolve(ivp,'t');
xf=@(t,a,b)eval(vectorize(x));
yf=@(t,a,b)eval(vectorize(y));
figure;hold on
t= -3:0.1:3;
for a= -2:2
    for b=-3:3
        plot(xf(t,a,b),yf(t,a,b))
    end
end
[X,Y]=meshgrid(-5:5,-5:5);
contour(X, Y, 3*X.^2 - 2*X*Y + Y.^2)
hold off
axis ([-5 5 -5 5])

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xlabel 'x'
ylabel 'y'
% the matrix id Linear source (unstable) lies inside the contour of the
% function: 3 x^2 - 2 x y + y^2

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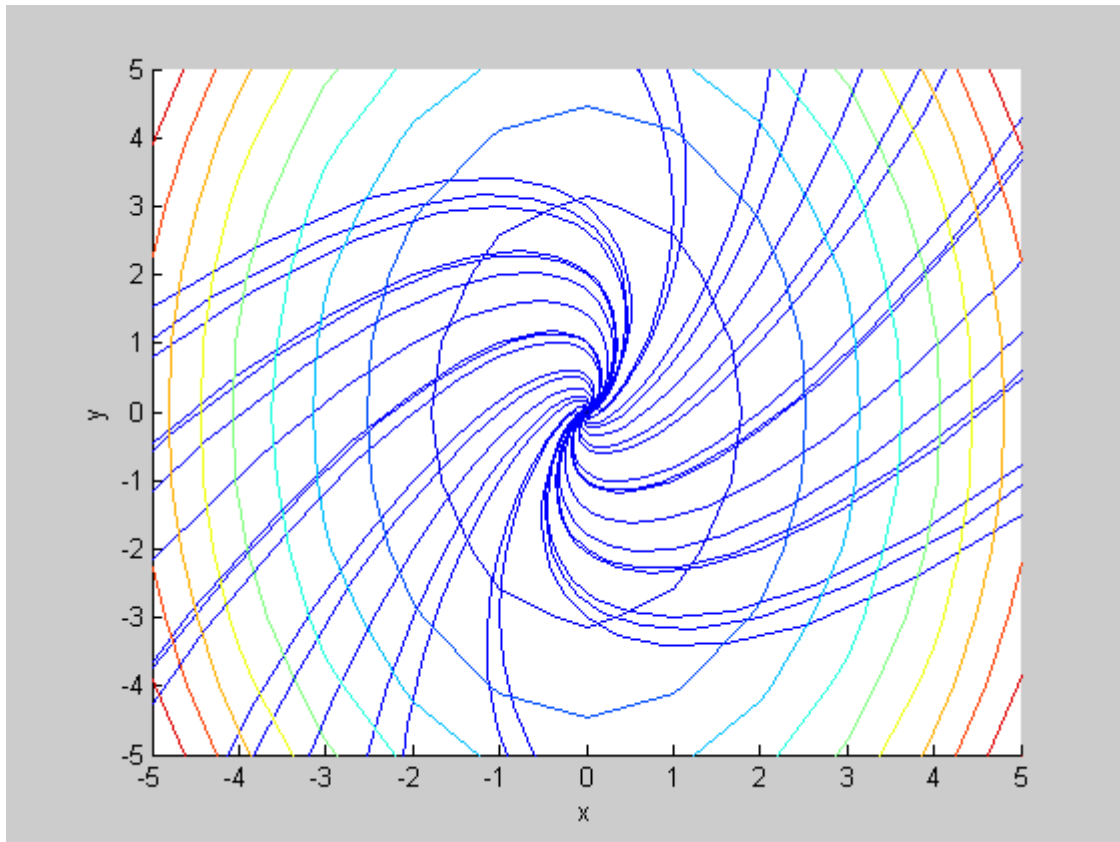
%E> m=2
ivp='Dx= 3*x-y,Dy=3*x+y,x(0)=a,y(0)=b';
[x,y]=dsolve(ivp,'t');
xf=@(t,a,b)eval(vectorize(x));
yf=@(t,a,b)eval(vectorize(y));
figure;hold on
t= -3:0.1:3;
for a= -2:2
    for b=-3:3
        plot(xf(t,a,b),yf(t,a,b))
    end
end
end

```

```

[X,Y]=meshgrid(-5:5,-5:5);
contour(X, Y, 3*X.^2 - 2*X*Y + Y.^2)
hold off
axis ([-5 5 -5 5])
xlabel 'x'
ylabel 'y'
% the matrix is the unstable Nodal source (repelling) lies inside the contour
of the
% function: 3 x^2 - 2 x y + y^2

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Conclusion:

At $m = -2$, the phase plane is Nodal Sink. As m increased to $-2^{1/2}$, the phase plane angled to the left and became Linear Sink. As m increased to 0, it became Saddle. With m increased to $2^{1/2}$, the phase plane became Linear Source. Finally, as m increased to 2, it became Nodal Source