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Math246 section 0221
Extra credit

A = **-2 ≤ μ ≤ 2**

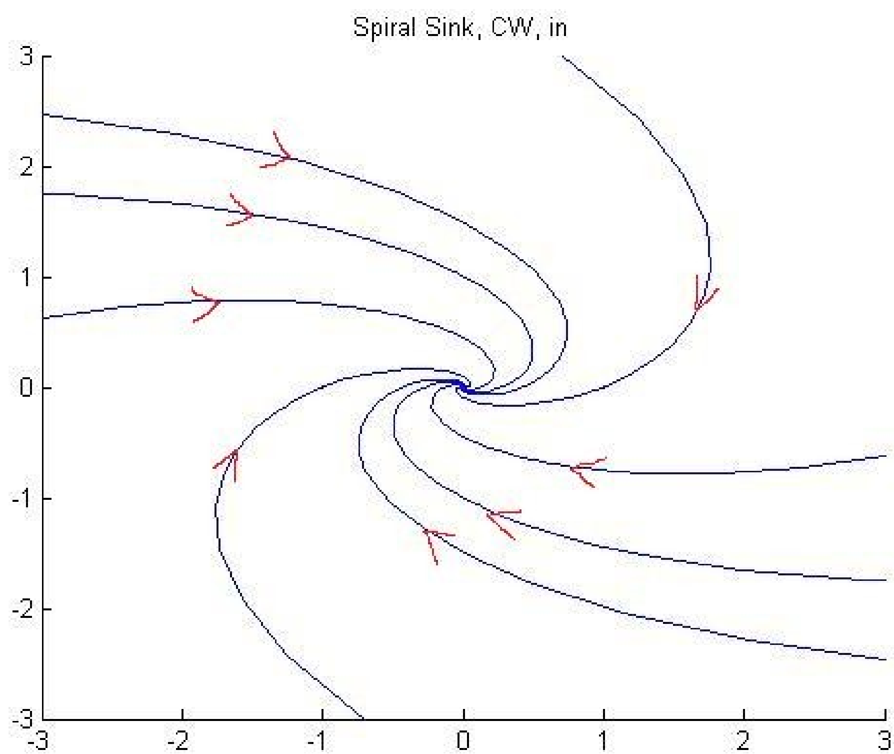
μ 1 - μ

1 + μ μ

Figure 1

μ = -2

```
EDU>> A = [-2 3;-1 -2]

A =

   -2     3
   -1    -2

EDU>> eig(A)

ans =

  -2.0000 + 1.7321i
  -2.0000 - 1.7321i

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Spiral Sink, CW, in'
```

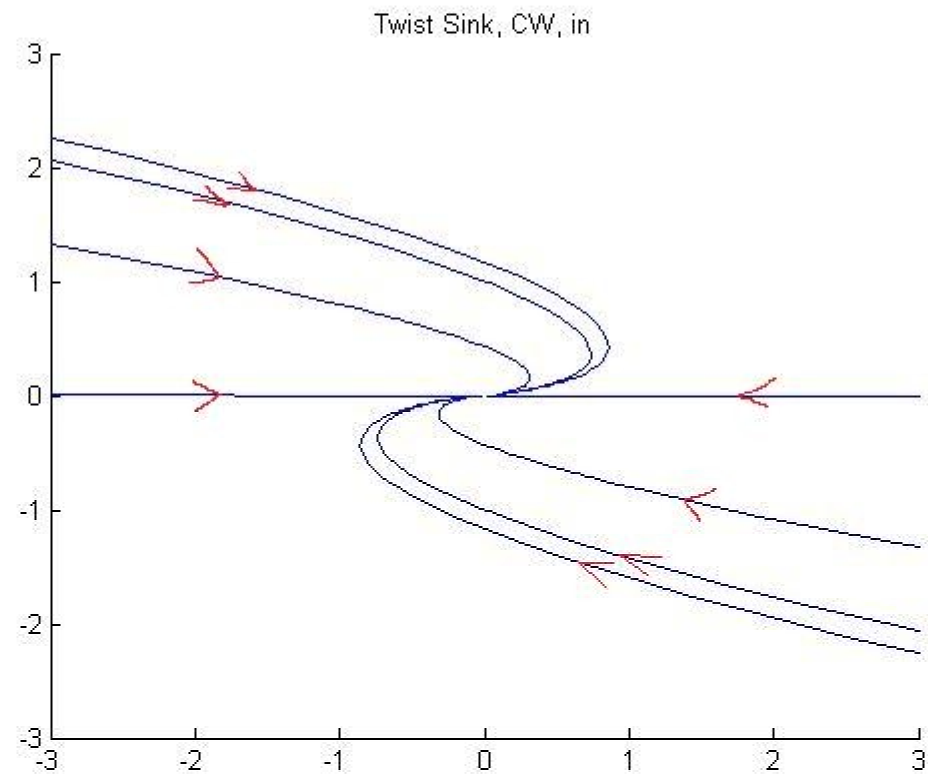


Mu=-2. The phase plane is a Spiral Sink. Since mu is positive, the origin is attracting. The spiral has clockwise rotation because a_21 is negative. The similar behavior of the phase plane occurs from mu=-2 to mu=-1.

Figure 2

μ = -1

```
EDU>> A = [-1 2;0 -1]

A =

   -1     2
    0    -1

EDU>> eig(A)

ans =

   -1
   -1

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Twist Sink, CW, in'
```

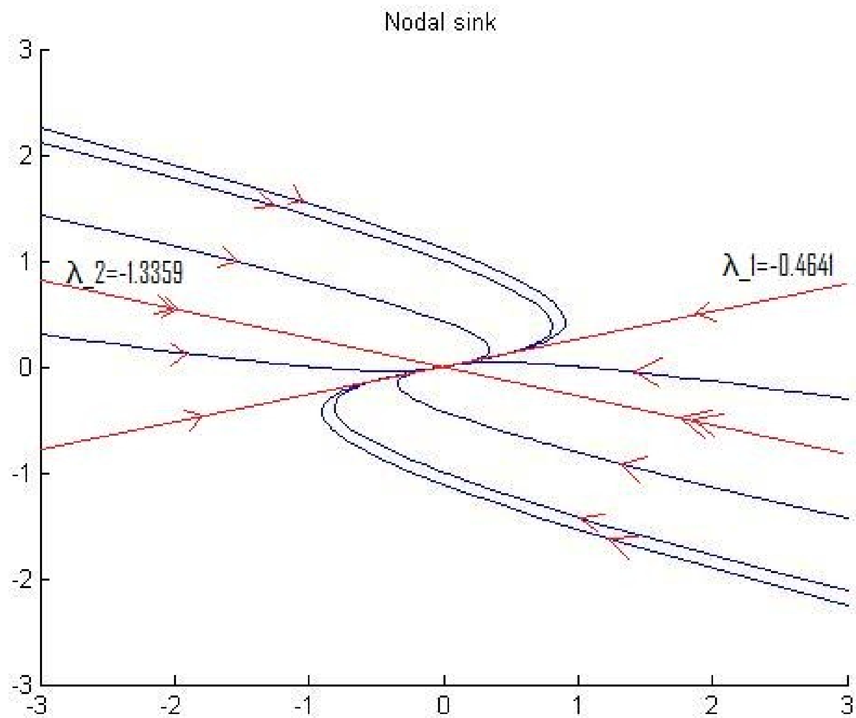


When mu=-1 the phase plane changes to a Twist Sink. Just to check, there are two Eigen values = -1 (multiplicity=2) and since A≠μI the graph is a Twist Sink. Since mu is negative, the origin is attracting. The spiral has clockwise rotation because a_12 is positive.

Figure 3

μ = -0.9

```
EDU>> A =[-0.9 1.9;0.1 -0.9]

A =

  -0.9000    1.9000
   0.1000   -0.9000

EDU>> [v,d]=eig(A)

v =

   0.9747   -0.9747
   0.2236    0.2236


d =

  -0.4641         0
        0   -1.3359

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Noddal sink'
```

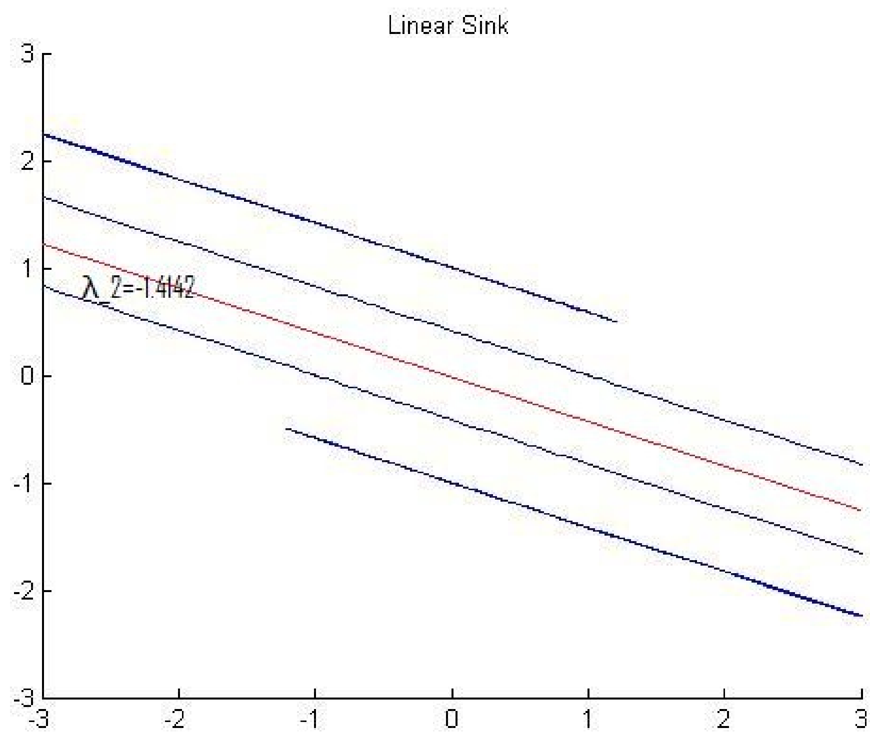


From mu=-1 to mu =-1/sqrt(2) the phase plane will look like a Nodal Sink (why mu goes to -1/sqrt(2) is explained in Figure 4). In this figure mu=-0.9 shows a typical behavior of the system. Since $\lambda\_2 \le \lambda\_1 \le 0$, the orbits will approach the origin tangent to the line λ_1. The origin is attracting because every orbit that starts near it will approach it as time goes to infinity.

Figure 4

μ = -1/sqrt(2) = -0.7071

```
EDU>> A=[-0.7071 1.7071;1-0.7071 -0.7071]

A =

  -0.7071    1.7071
   0.2929   -0.7071

EDU>> [v,d]=eig(A)

v =

   0.9239   -0.9239
   0.3827    0.3827


d =

   0.0000         0
        0   -1.4142

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Linear Sink'
```

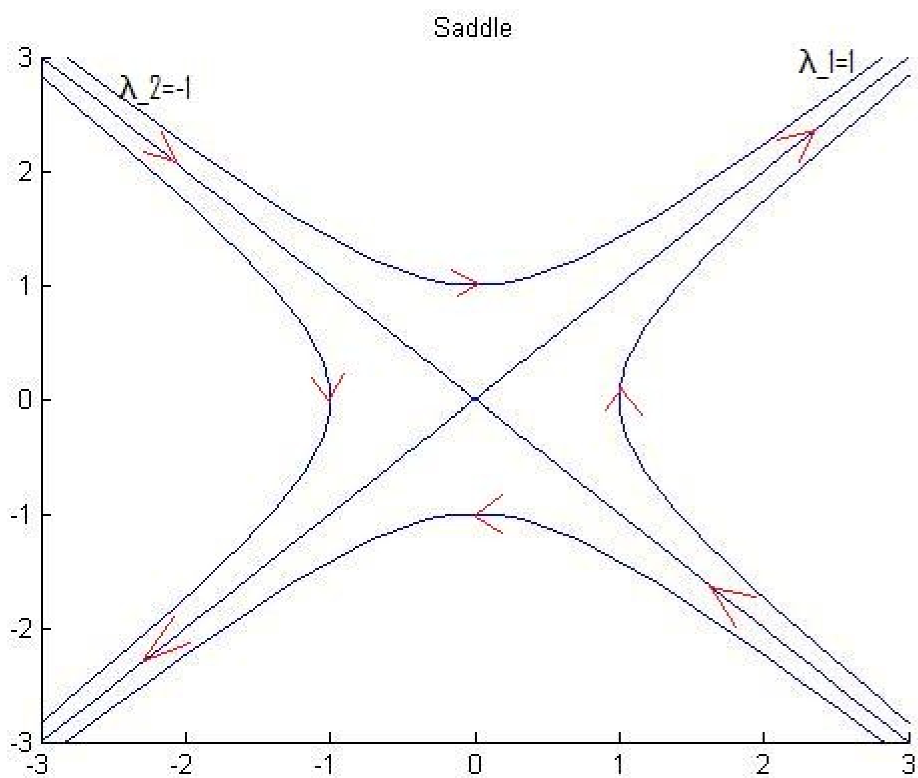


A Linear Sink occurs then det(A)=0. From matrix A, det(A)=μ^2-(1+μ)*(1- μ)=0; 2* μ ^2=1; μ=±1/sqrt(2). Therefore, the phase portrait transforms from Nodal Sink to Linear Sink. One Eigen value is -1.4142. The other is 0 and it is a line of stationary points. As time increases the solution will approach one of those stationary points as along a line that is parallel to the line λ_2. All orbits not on the line of stationary points will approach that line parallel λ_2. The origin is stable but not attracting.

Figure 5

μ = 0

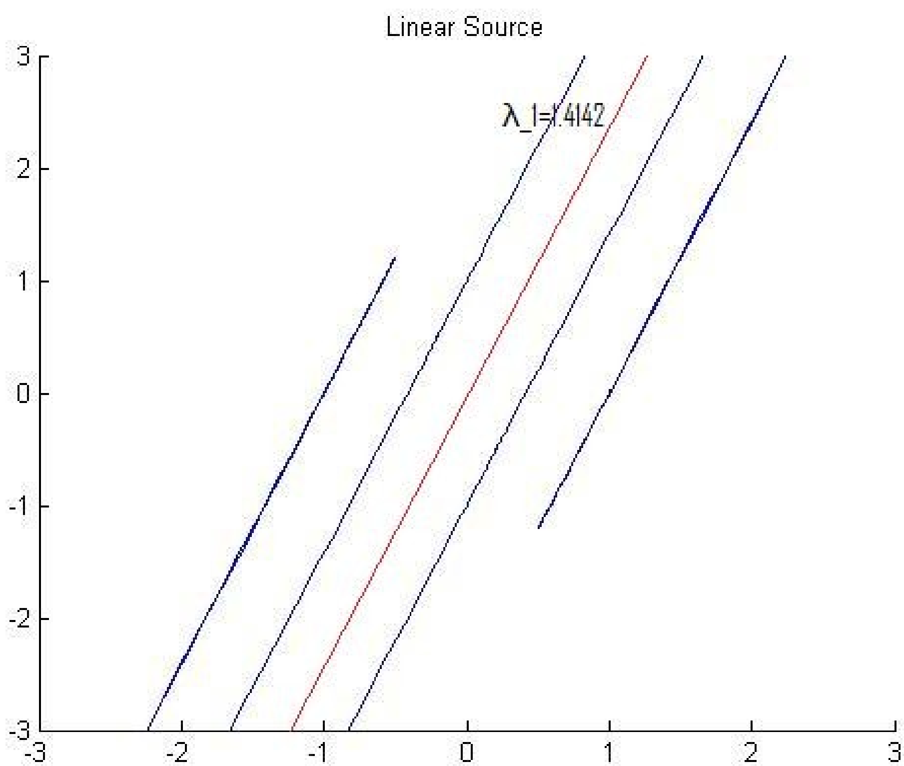
```
EDU>> A=[0 1;1 0]

A =

     0     1
     1     0

v =

   -0.7071    0.7071
    0.7071    0.7071


d =

    -1     0
     0     1

EDU>> figure, hold on
for j=1:8
    [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
    plot(y(:,1), y(:, 2))
    [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
    plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Saddle'
```



As mu goes from =-1/sqrt(2) to 0 it is a saddle and continues to be a saddle, just on a tilt in the other direction until mu is =+1/sqrt(2). In this figure mu = 0, so it is not tilted. Since λ_2≤0≤ λ_1, the phase portrait is a Saddle. Eigen values are real. The origin is unstable but not repelling.

μ = 1/sqrt(2) = 0.7071

```
EDU>> A=[0.7071 1-0.7071;1.7071 0.7071]

A =

   0.7071    0.2929
   1.7071    0.7071

EDU>> [v,d]=eig(A)

v =

   0.3827   -0.3827
   0.9239    0.9239


d =

   1.4142         0
        0   -0.0000

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Linear Source'
```

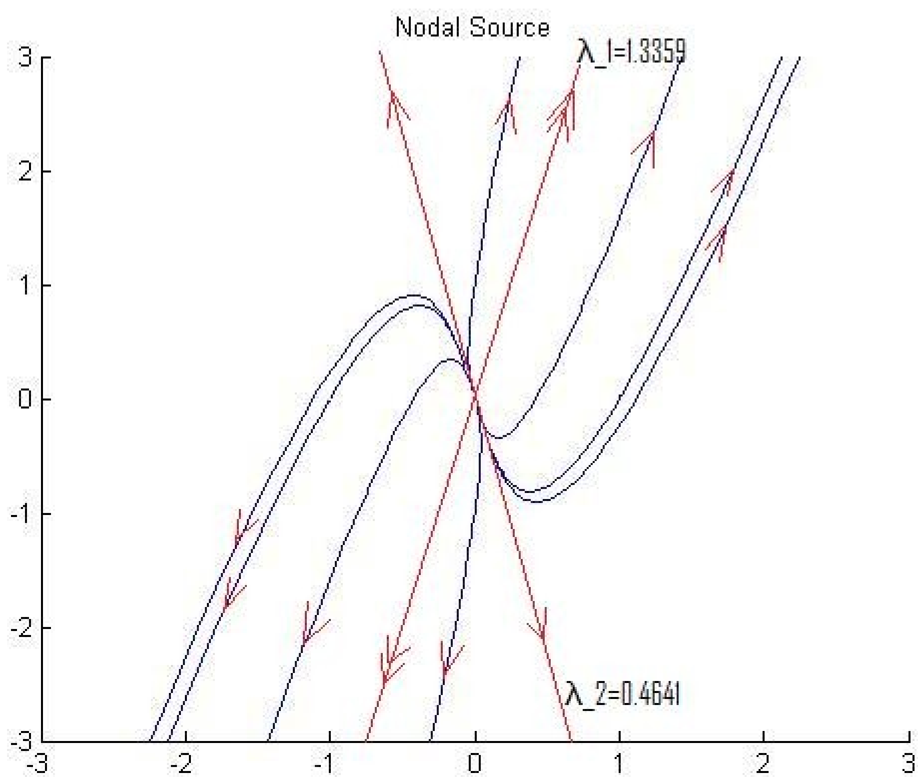


The phase portrait transforms from Saddle to Linear Source when mu = +1/sqrt(2). One Eigen value is 1.4142. The other is 0 and it is a line of stationary points. As time increases the solution will approach one of those stationary points as along a line that is parallel to the line λ_1. All orbits not on the line of stationary points will approach that line parallel λ_1. The origin is unstable but not repelling.

Figure 7

μ = 0.9

```
EDU>> A=[0.9 0.1;1.9 0.9]

A =

   0.9000    0.1000
   1.9000    0.9000

EDU>> [v,d]=eig(A)

v =

   0.2236   -0.2236
   0.9747    0.9747


d =

   1.3359         0
        0    0.4641

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Nodal Source'
```

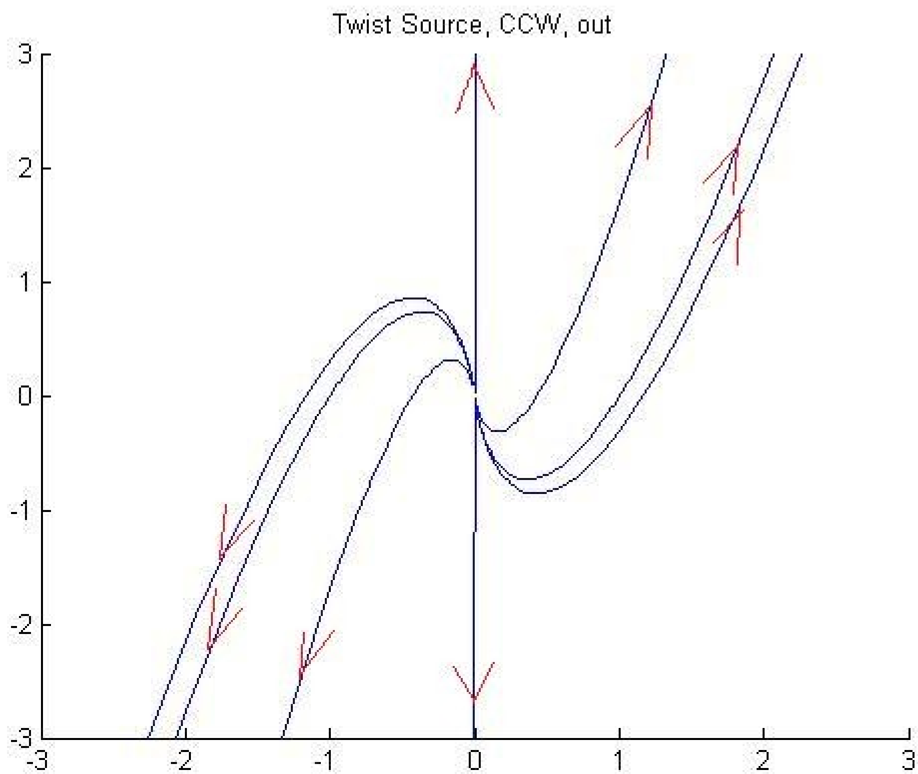


From mu=+1/sqrt(2) to mu =1 the phase plane will look like a Nodal Source. In this figure mu=0.9 shows a typical behavior of the system. Since $0 \le \lambda_2 \le \lambda_1$, the orbits will approach the origin tangent to the line λ_2. The origin is repelling because every orbit that starts near it will approach away from it as time goes to infinity.

Figure 8

μ = 1

```
EDU>> A=[1 0;2 1]

A =

    1    0
    2    1

EDU>> eig(A)

ans =

    1
    1

EDU>> figure, hold on
for j=1:8
   [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
   [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
   plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Twist Source, CCW, out'
```

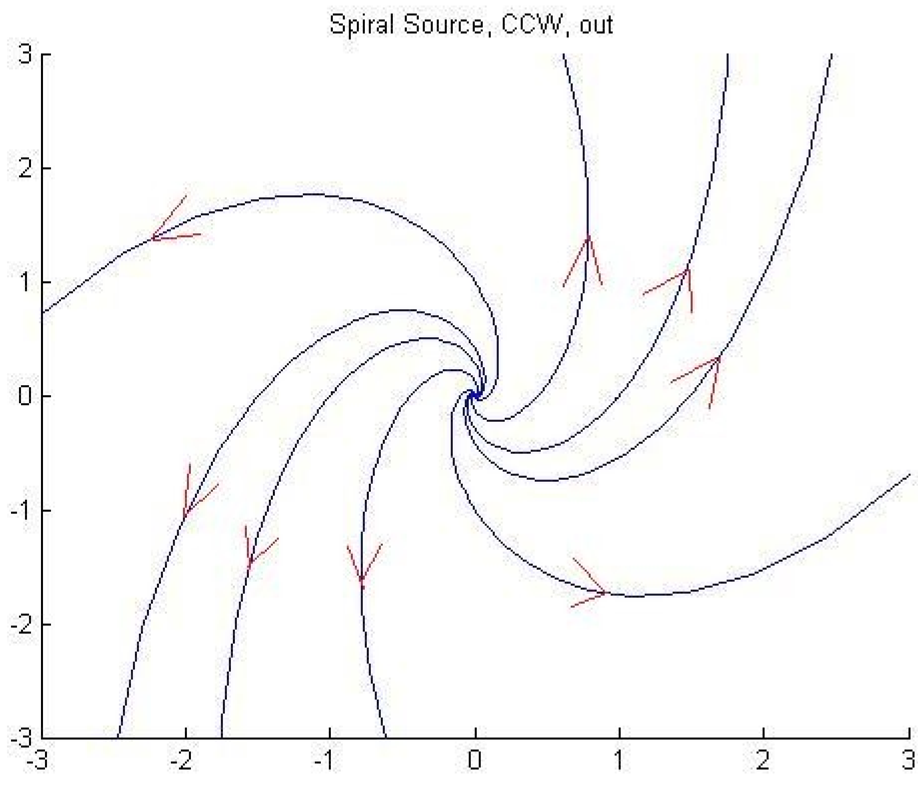


When mu=1 the phase plane changes to a Twist Source. There are two Eigen values = 1 (multiplicity=2) and since A≠μI the graph is a Twist Source. Since mu is positive, the origin is repelling. The spiral has counterclockwise rotation because a_21 is positive.

Figure 9

μ = 2

```
EDU>> A=[2 -1;3 2]

A =

     2    -1
     3     2

EDU>> eig(A)

ans =

   2.0000 + 1.7321i
   2.0000 - 1.7321i

EDU>> figure, hold on
for j=1:8
    [t, y] = ode45(@(t, y) A*y, [0, 4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
    plot(y(:,1), y(:, 2))
    [t, y] = ode45(@(t, y) A*y, [0, -4], [cos(2*j*pi/8), sin(2*j*pi/8)]);
    plot(y(:,1), y(:, 2))
end
axis([-3,3,-3,3]), hold off
EDU>> title 'Spiral Source, CCW, out'
```



From mu=1 to mu=2 the phase plane is a Spiral Source. Since mu is positive, the origin is repelling. The spiral has counterclockwise rotation because a_21 is positive.