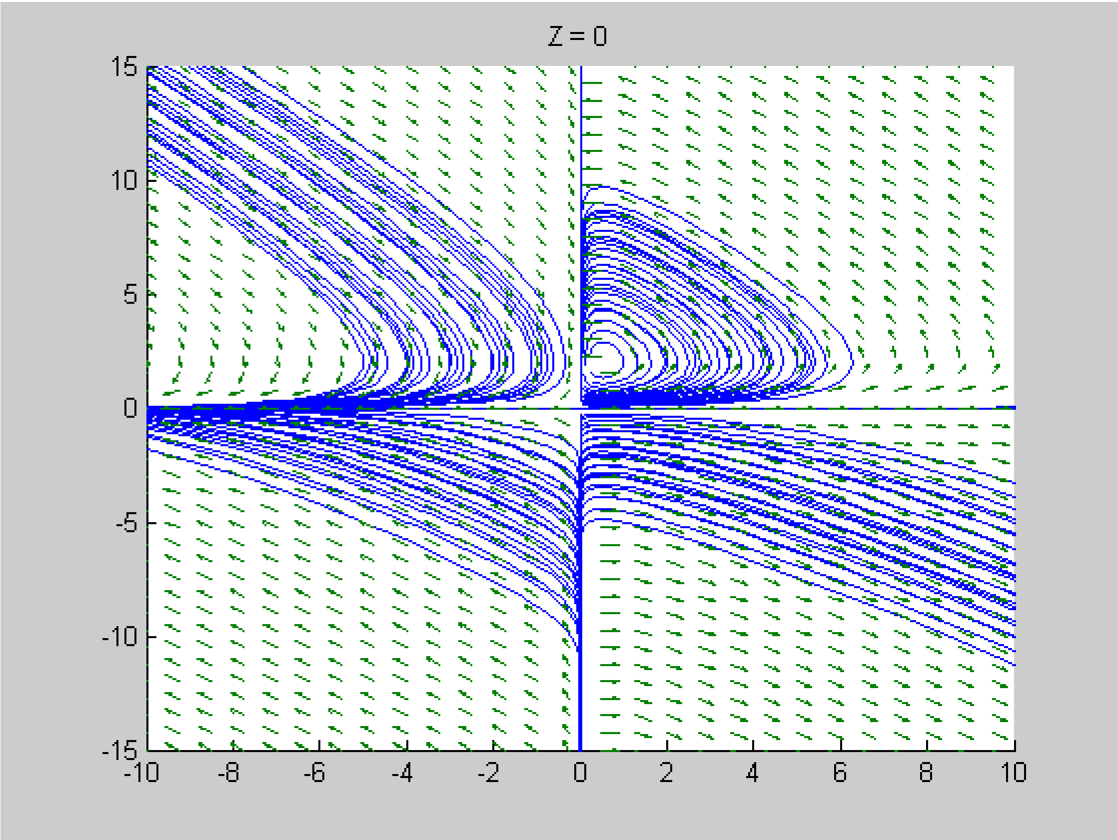
Transition from #2 to #3 from chapter 9.5 - Chris Palm (Bossard 11am)

Z = 0

```
warning off all
syms x y q Z; Z = 0;
q = 5;

figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -q:q
    for b = -q:q
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0:'); disp([xc yc])

A = jacobian([sys1 sys2], [x y])
evals = eig(A);

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (1/2,2):');
disp(double(subs(evals, {x y}, {(1/2), 2})))

% (0,0) is saddle point and is unstable.
% (1/2, 2) is a centers and is in the ccw direction and stable.

% As there becomes more predators(y becomes larger)
% the prey decreases(x becomes smaller).  This is a continuous cycle.  Once
% there becomes more prey, the predators become abundant and so therefore
% the prey deminishes.
```

```
Critical points for Z = 0:
[   0,    0]
[ 1/2,    2]


A =

[    1-1/2*y,      -1/2*x]
[      1/2*y, -1/4+1/2*x]
```

```
Eigenvalues at (0,0):
    1.0000
   -0.2500

Eigenvalues at (1/2,2):
   0.0000 + 0.5000i
  -0.0000 - 0.5000i
```

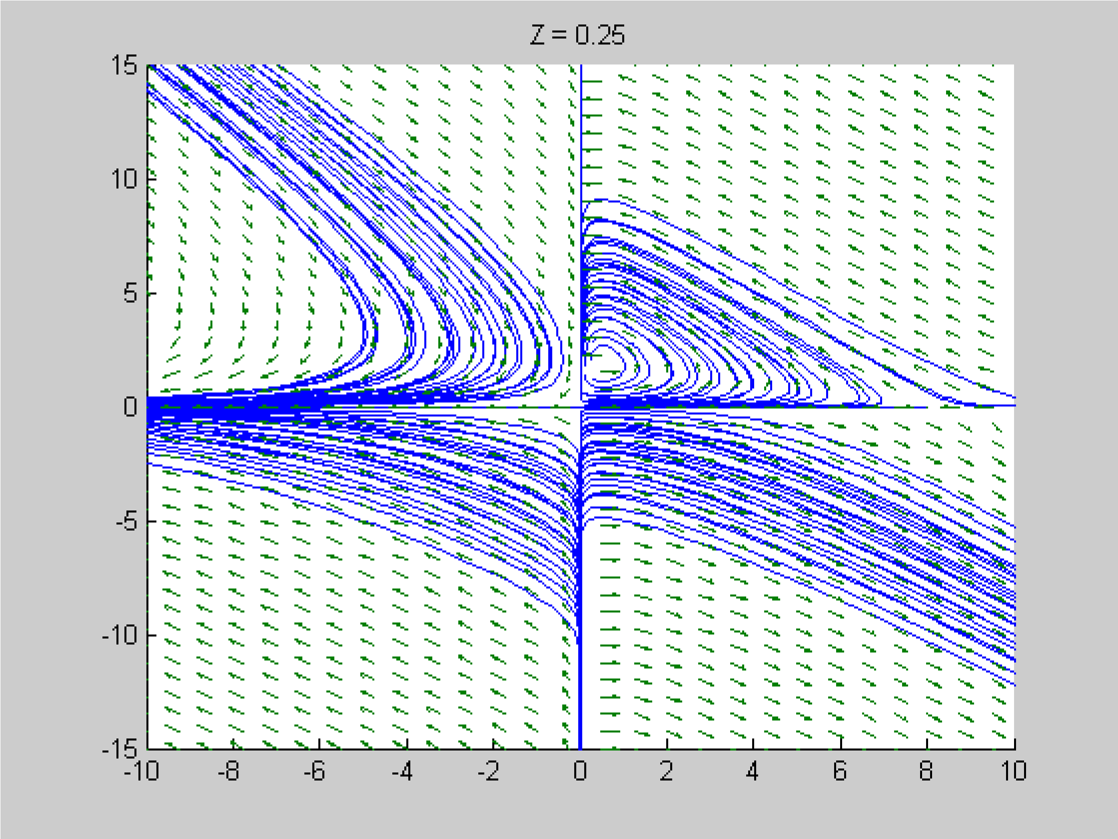


Z = 0.25

```
warning off all
syms x y Z; Z = 0.25;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0.25'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0.25:'); disp([xc yc])

A = jacobian([sys1 sys2], [x y])
```

```
evals = eig(A);

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (8,0):');
disp(double(subs(evals, {x y}, {8, 0})))
disp('Eigenvalues at (1/2,15/8):');
disp(double(subs(evals, {x y}, {(1/2), (15/8)})))

% (0,0) continues to be an unstable saddle point.
% (8,0) is also an unstable saddle point.
% (1/2, 15/8) is still a centers and in the ccw direction and stable.

% The predator still depends on the prey, but has a slightly more difficult
time
% reproducing because of the saddle point at (8,0).  This difficulty is
% only present for larger values of y.
```

```
Critical points for Z = 0.25:
[    0,    0]
[    8,    0]
[  1/2, 15/8]


A =

[ 1-1/2*y-1/4*x,          -1/2*x]
[         1/2*y,     -1/4+1/2*x]


Eigenvalues at (0,0):
    1.0000
   -0.2500

Eigenvalues at (8,0):
    3.7500
   -1.0000

Eigenvalues at (1/2,15/8):
  -0.0312 + 0.4831i
  -0.0313 - 0.4831i
```

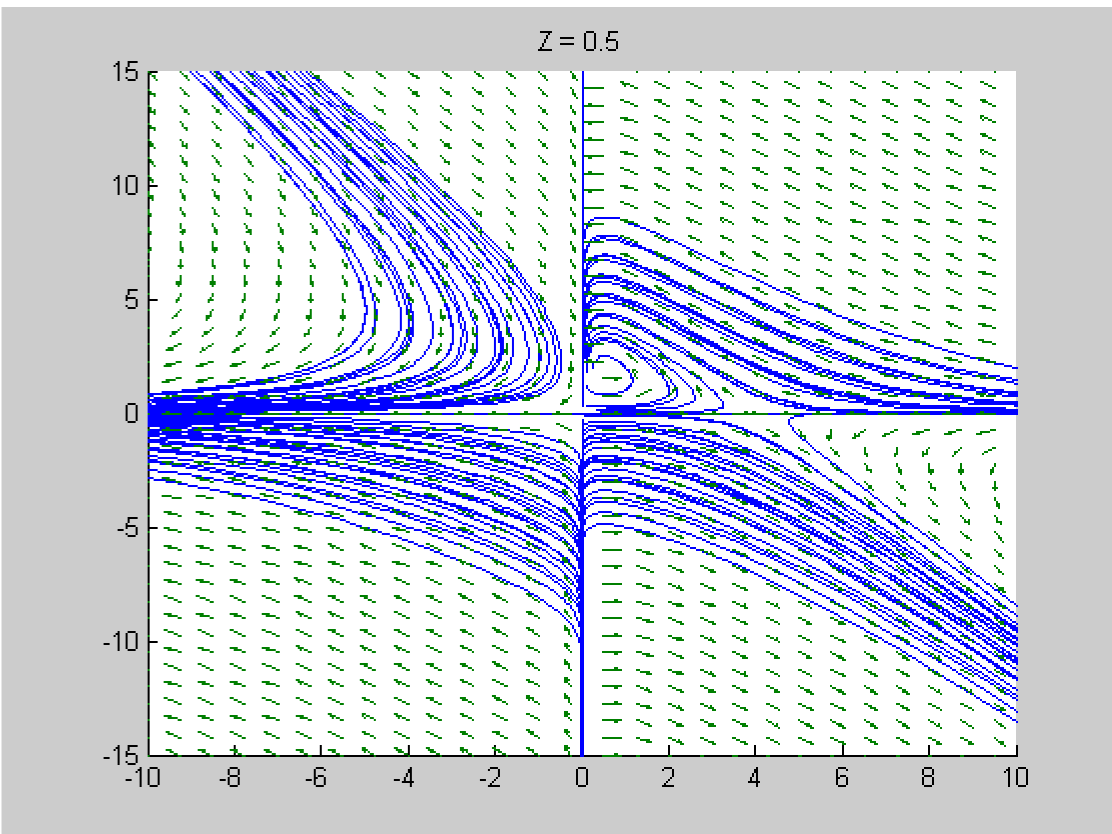


Z = 0.5

```
warning off all
syms x y Z; Z = 0.5;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0.5'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0.5:'); disp([xc yc])

A = jacobian([sys1 sys2], [x y])
evals = eig(A);

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (4,0):');
disp(double(subs(evals, {x y}, {4, 0})))
disp('Eigenvalues at (1/2,7/4):');
disp(double(subs(evals, {x y}, {(1/2), (7/4)})))

% (0,0) is still an unstable saddle point.
```

```
% (4,0) is also an unstable saddle point.
% (1/2, 7/4) is a ccw centers and is stable.

% In the predator prey model, the predator(y) depends on x.  It is a
% continuous cycle and the prey does not have a difficult time reproducing
% for smaller values of y.
```

```
Critical points for Z = 0.5:
[   0,   0]
[   4,   0]
[ 1/2, 7/4]


A =

[ 1-1/2*y-1/2*x,        -1/2*x]
[         1/2*y,    -1/4+1/2*x]


Eigenvalues at (0,0):
    1.0000
   -0.2500

Eigenvalues at (4,0):
    1.7500
   -1.0000

Eigenvalues at (1/2,7/4):
  -0.0625 + 0.4635i
  -0.0625 - 0.4635i
```

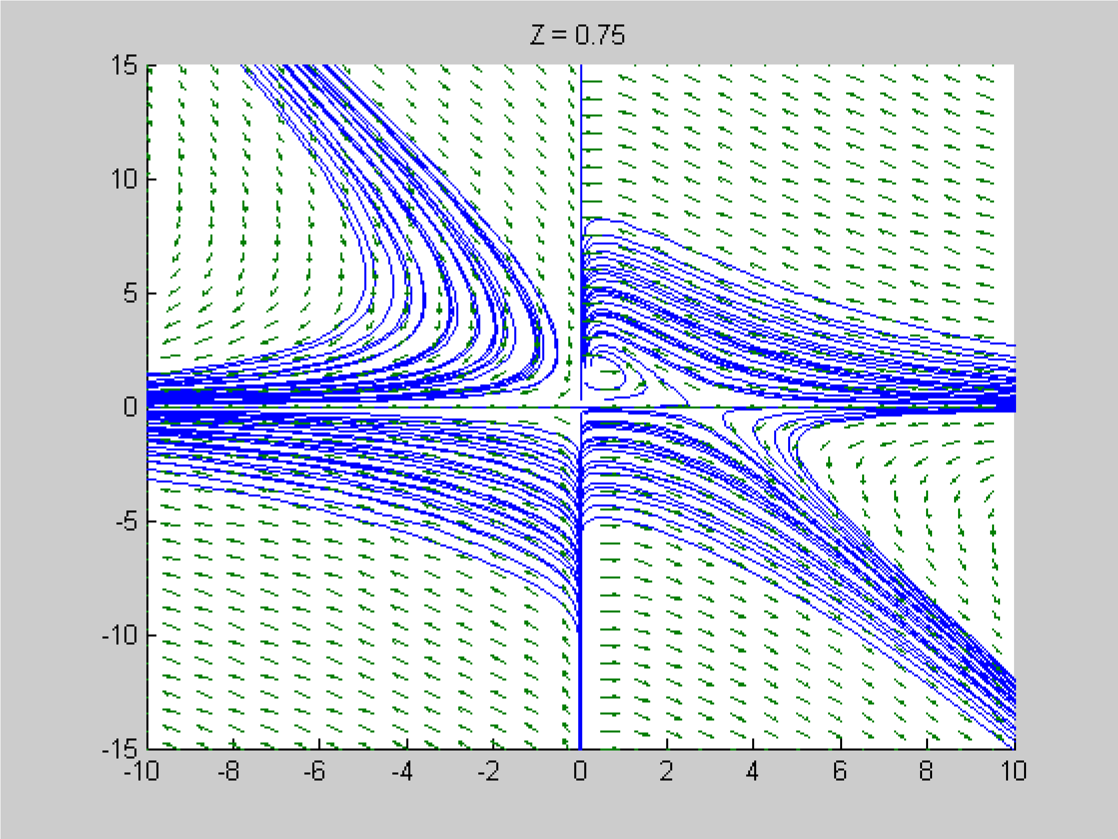


Z = 0.75

```
warning off all
syms x y Z; Z = 0.75;
```

```
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 0.75'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 0.75:'); disp([xc yc])

A = jacobian([sys1 sys2], [x y])
evals = eig(A);

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (8/3,0):');
disp(double(subs(evals, {x y}, {(8/3), 0})))
disp('Eigenvalues at (1/2,13/8):');
disp(double(subs(evals, {x y}, {(1/2), (13/8)})))

% (0,0) is an unstable saddle point.
% (8/3,0) is an unstable saddle point.
% (1/2, 13/8) is a stable, ccw centers.

% Here the prey(x) has more difficulty in reproducing once the prey has
% deminished the prey's population.  This is due to the fact that the
% centers point is nearing the unstable saddle at (0,0) and the saddle
% point at (8/3, 0)
```

```
Critical points for Z = 0.75:
[    0,    0]
[  8/3,    0]
[  1/2, 13/8]


A =

[ 1-1/2*y-3/4*x,        -1/2*x]
[         1/2*y,    -1/4+1/2*x]


Eigenvalues at (0,0):
    1.0000
   -0.2500

Eigenvalues at (8/3,0):
    1.0833
   -1.0000

Eigenvalues at (1/2,13/8):
  -0.0937 + 0.4408i
  -0.0938 - 0.4408i
```

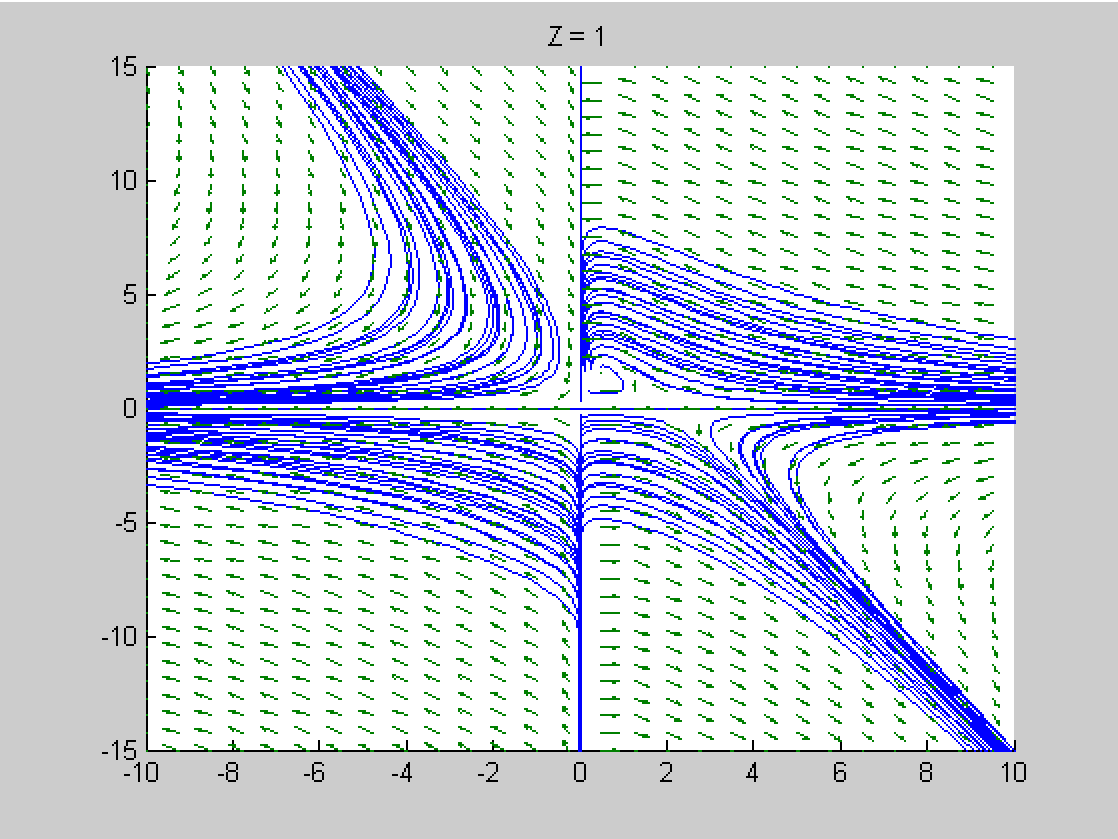


Z = 1

```
warning off all
syms x y Z; Z = 1;
figure; hold on
f = @(t, x) [x(1)*(1 - (1/2)*x(2) - (Z/2)*x(1)); x(2)*((-1/4) + (1/2)*x(1))];
for a = -5:5
    for b = -5:5
        [t, xa] = ode45(f, [0 5], [a b]);
        plot(xa(:,1), xa(:,2))
        [t, xa] = ode45(f, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
    end
end
title 'Z = 1'
[X, Y] = meshgrid(-10:0.75:10, -15:0.75:15);
U = X.*(1 - (1/2)*Y - (Z/2)*X);
V = Y.*((-1/4) + (1/2)*X);
L = sqrt((U).^2 + (V).^2);
quiver(X, Y, U./L, V./L, 0.4);
axis([-10 10 -15 15])

sys1 = x*(1 - (1/2)*y - (Z/2)*x);
sys2 = y*((-1/4) + (1/2)*x);
[xc, yc] = solve(sys1, sys2, x, y);
disp('Critical points for Z = 1:'); disp([xc yc])

A = jacobian([sys1 sys2], [x y])
evals = eig(A);

disp('Eigenvalues at (0,0):');
disp(double(subs(evals, {x y}, {0, 0})))
disp('Eigenvalues at (2,0):');
disp(double(subs(evals, {x y}, {2, 0})))
disp('Eigenvalues at (1/2,3/2):');
disp(double(subs(evals, {x y}, {(1/2), (3/2)})))

% (0,0) is an unstable saddle.
```

```
% (2,0) is an unstable saddle.
% (1/2, 3/2) is a stable ccw centers.

% In this model the predator(y) still depends on x.  The prey(x) has a very
% difficult time reproducing once it nears 0 because of how close the
% centers is to the origin(an unstable saddle) and the saddle point at
% (2,0).
```

```
Critical points for Z = 1:
[   0,   0]
[   2,   0]
[ 1/2, 3/2]


A =

[  1-1/2*y-x,      -1/2*x]
[      1/2*y, -1/4+1/2*x]


Eigenvalues at (0,0):
    1.0000
   -0.2500

Eigenvalues at (2,0):
    0.7500
   -1.0000

Eigenvalues at (1/2,3/2):
  -0.1250 + 0.4146i
  -0.1250 - 0.4146i
```



Analysis

```
% (0,0) is a critical point for all graphs, and is a saddle point.
```

```
% Therefore the origin is unstable for all the graphs.

% When Z = 0, the other critical point is a ccw centers.  Once Z > 0, this
% critical point becomes a spiral sink and continues to be stable and ccw.  Its
% x-coordinate is 1/2 for 0=>Z=>1 and the y-coordinate decreases as Z --> 1.

% The third critical point appears once Z > 0.  It is a saddle point also,
% like (0,0) and has a constant y-coordinate of 0, while its x-coorindate
% decreases as Z --> 1.

% For 0=>Z=>1, a_21 > 0 so all critical points are in the ccw direction.

% y is the predator and x is the prey in this predator-prey system.  For Z
% = 0, as there becomes more predators(y becomes larger) and so therefore
% the prey decreases(x becomes smaller).  This is a continuous cycle.  Once
% there becomes more prey, the predators become abundant and so therefore
% the prey deminishes.

% This pattern continues as Z increases towards 1.  When Z = 1, y(the
% predator) still depends on x(the prey), but now the prey(x) decreases as
% y increases along with when x increases.  The addition of the saddle point
along the x-axis causes this.
% The predator(y) becomes larger
% when x becomes smaller because the predators are eating all of their
% food (x).

% The predator-prey model is a continuous loop of events(prey
% becomes abundant, predators eat most of prey, predators decrease in
% population until the prey becomes abundant again), while Z is closer to
% 0. As Z = 1, the prey do not reproduce for large values of y(predators).
```