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%Kevin Paszinski 5/12/09 Math246 0231
%Extra Credit Matlab Assignment
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%Attached to this PDF file is a scanned copy of the written work done with

%this assignment. The original problem given was

%(d^2x)/(dt^2) + b(dx/dt) + x - x^2 = 0 where b=[0,1]

%The scanned sheet shows the reduction of the problem into a first order

%equation (dx/dt and dy/dt). From here I found the stationary points by

%setting the derivative equal to zero. The stationary points came to (0,0)

%and (1,0). Linearization was done to find that (0,0) was a CW center and

%(1,0) was a saddle point. When graphed you get the phase portraits below.

%I printed the graphs for b=0 to b=1 on incriments of 0.1 to show the small

%changes as b increased. I graphed b=0 twice to start to show a large view

%of the portrait to make sure nothing was going on with large values of t.

%The next 11 portraits are all zoomed in real close to show the slight

%changes at the stationary points as b increases.

%There is a perfectly defined saddle and center at b=0. As b increases, the

%portrait becomes distorted as the first derivative of x is being factored

%into the equation. This is causing the graph to be stretched approximatly

%on the line y=-x. This deformation eventually causes the center to become

%an ellipse and then break open somewhere between b= 0.8 and 0.9. The

%saddle keeps its main shape half way through. When b approaches 0.8 the

%right hand side of the saddle remains intact but the saddle breaks on the

%left hand side opening up to the broken side of the center forming a gap

%in the center of the portrait. When b finally reaches a value of 1, the

%center of the graph finally stops distorting. The portrait seems to only

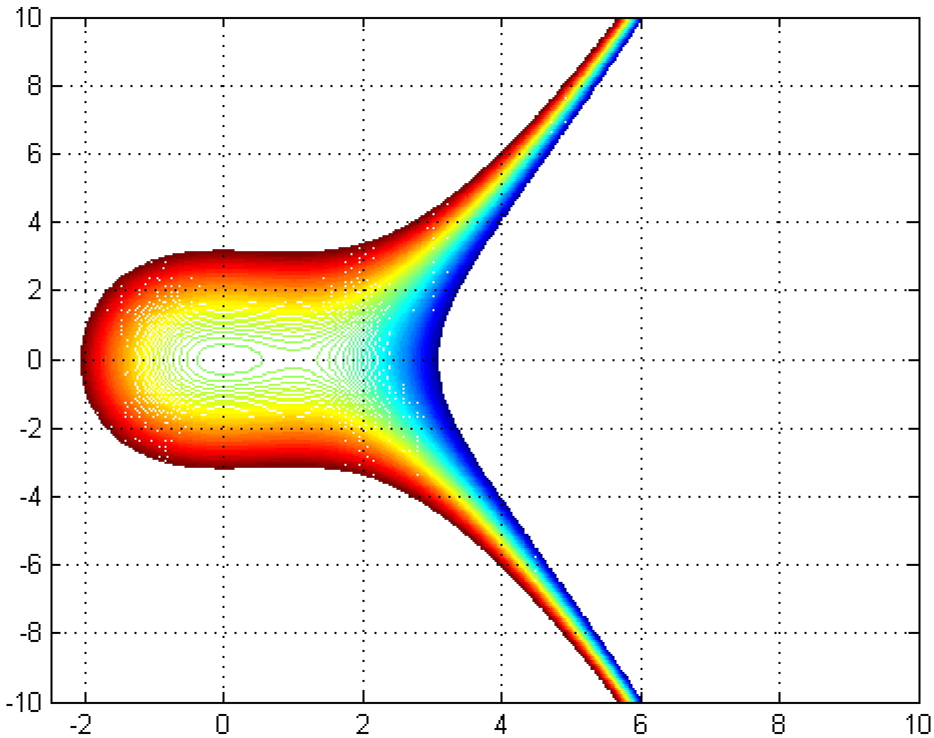
%revolve clockwise around (0,0).

%In conclusion the behavior of this second order differential equation

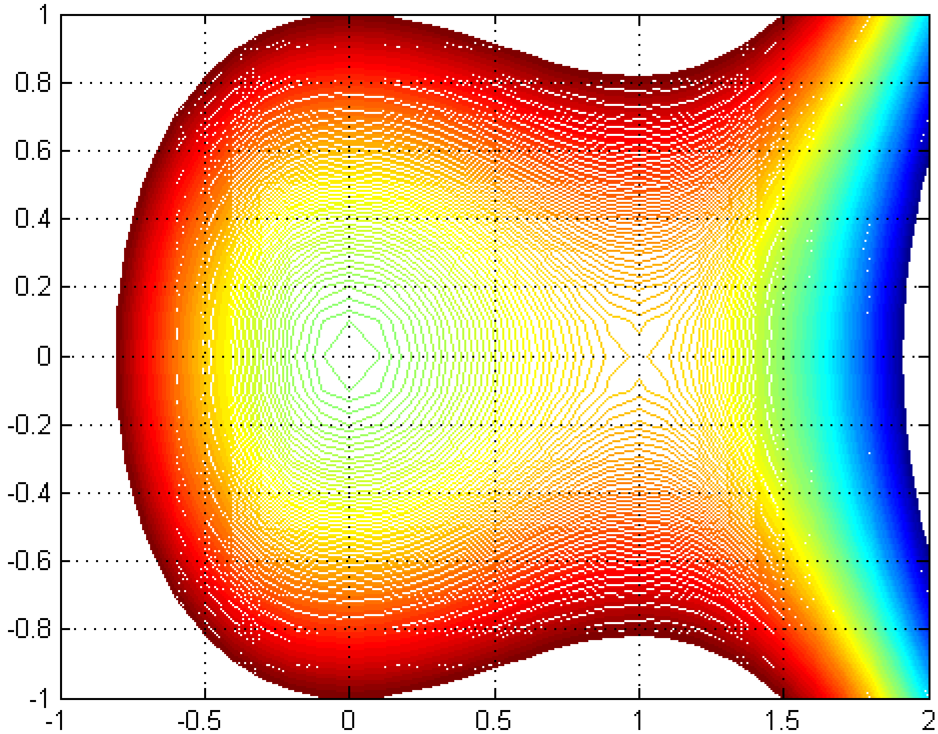
%doesn't change drastically until higher values of b. From here one

%stationary point is lost and another is redefined.

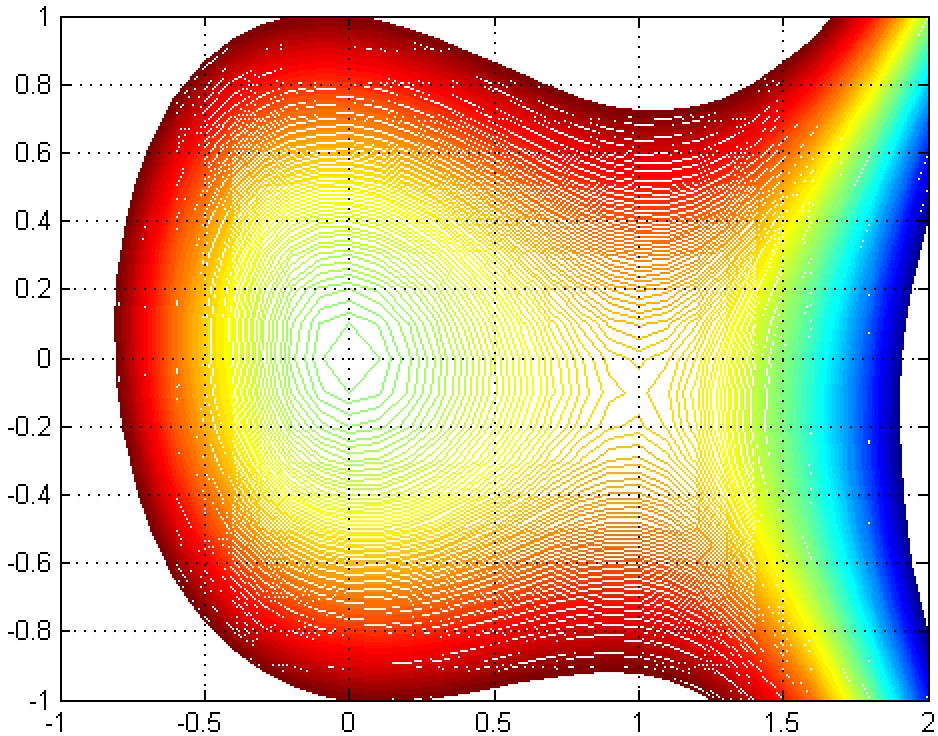
```
b=0;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-5:.1:5;

contour(X,Y,Z,level)

axis([-2.5 10 -10 10])

grid on
```



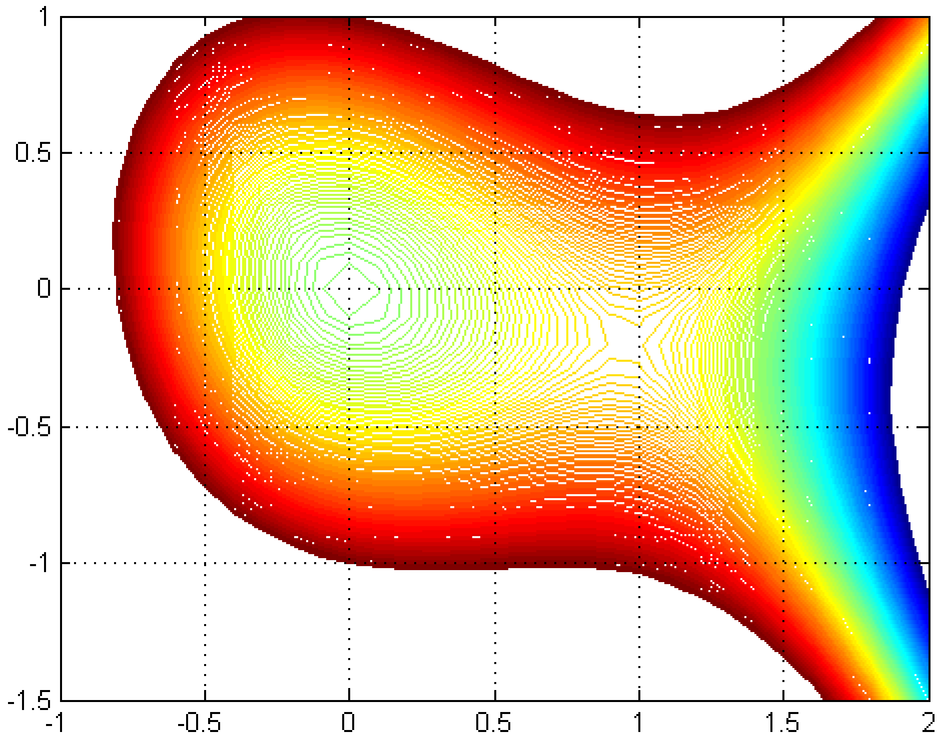

```
b=0;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1 1])

grid on
```



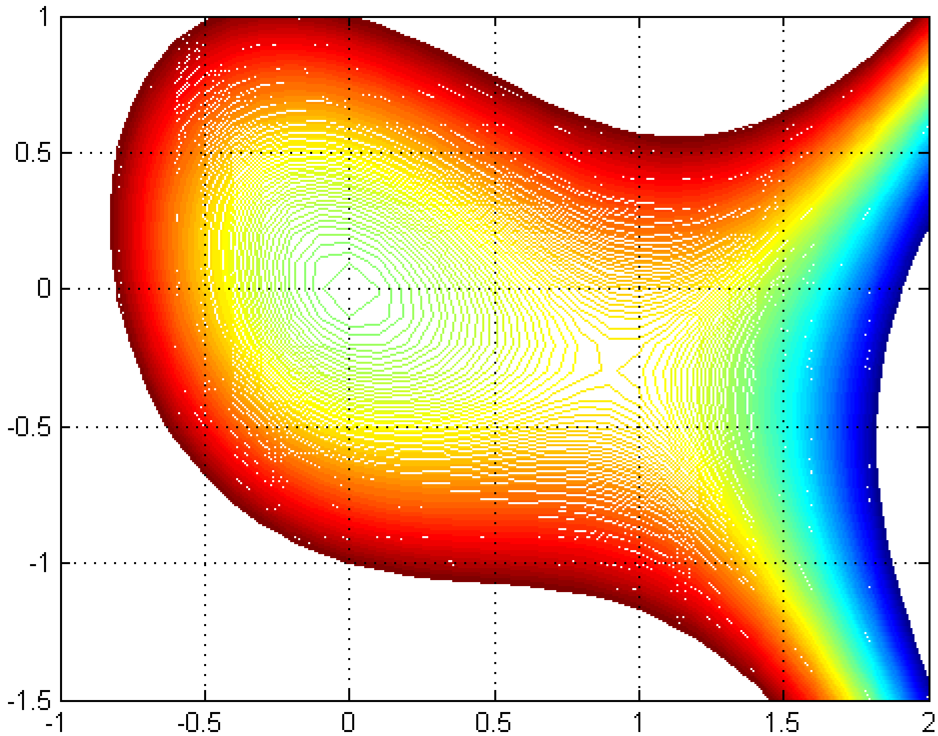

```
b=0.1;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1 1])

grid on
```



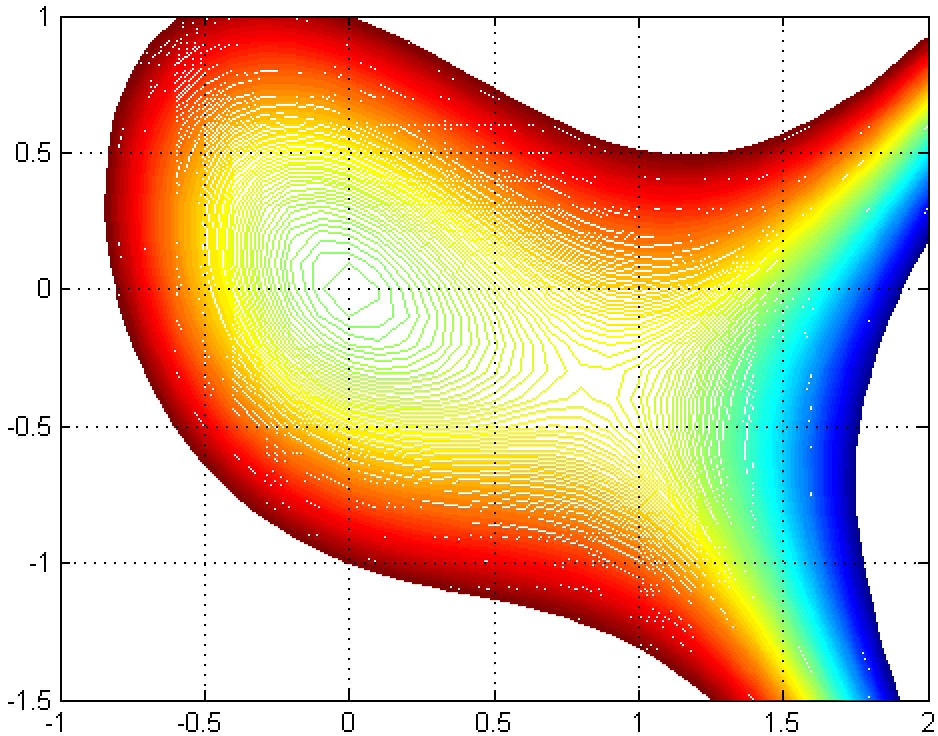

```
b=0.2;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1.5 1])

grid on
```



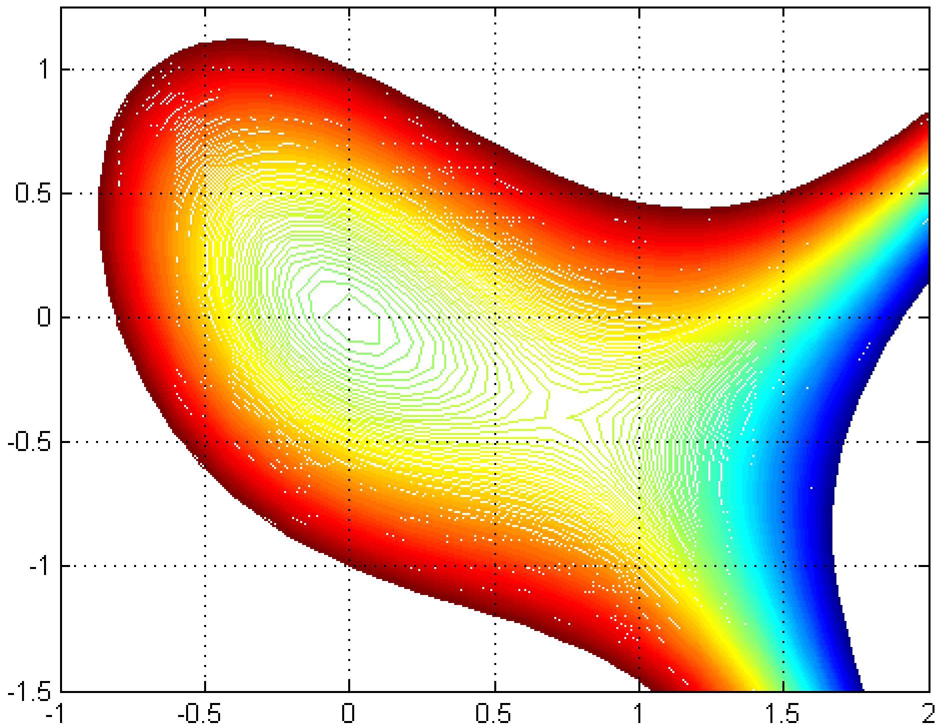

```
b=0.3;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1.5 1])

grid on
```



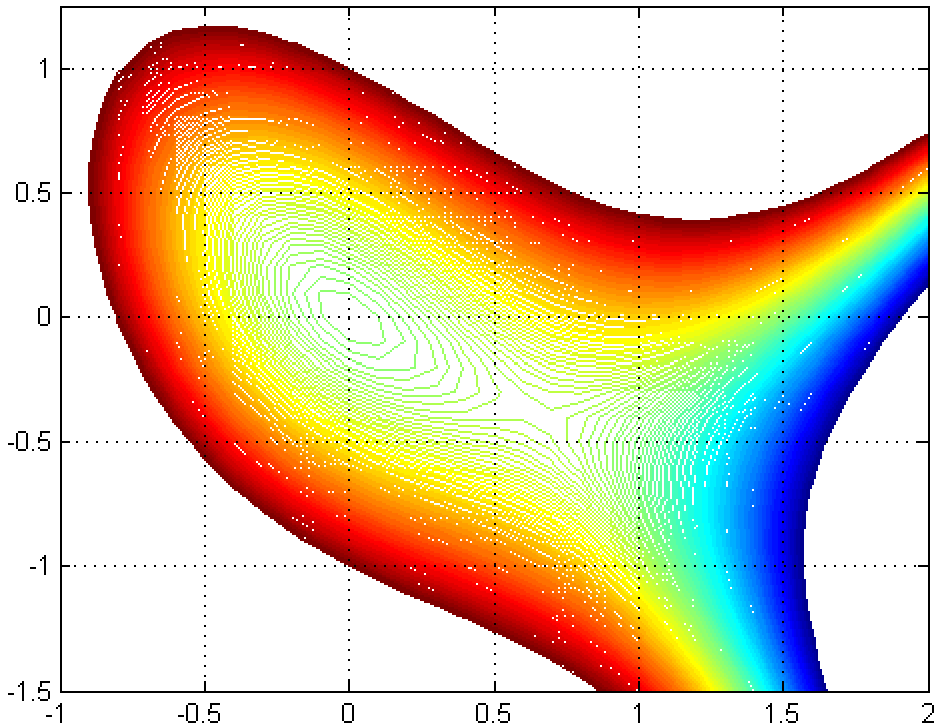

```
b=0.4;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1.5 1])

grid on
```



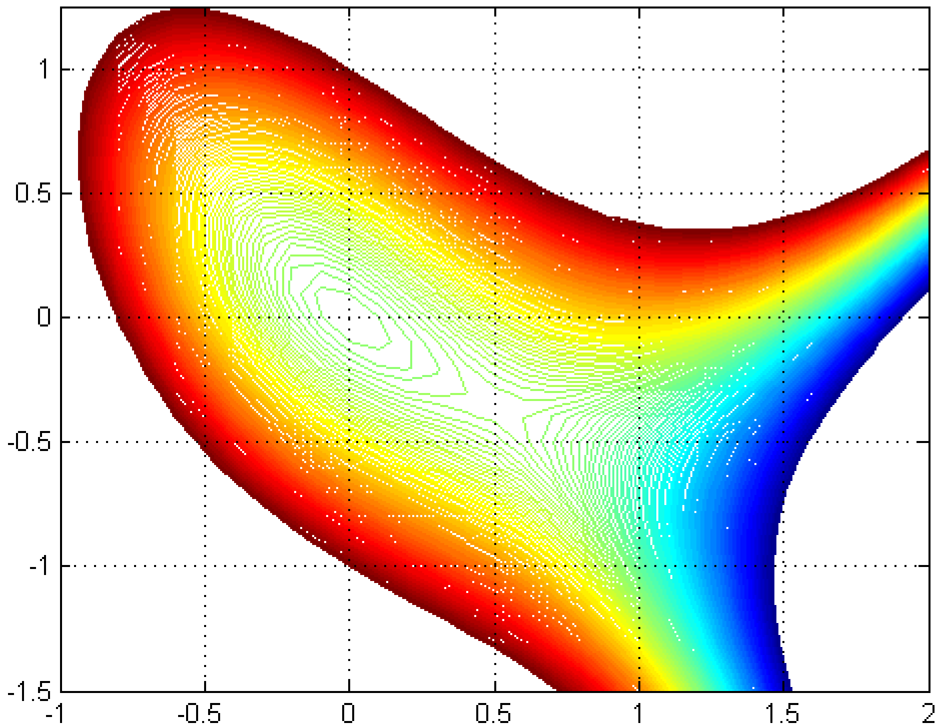

```
b=0.5;
f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;
x=-15:.1:15; y=-15:.1:15;
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
level=-.5:.005:.5;
contour(X,Y,Z,level)
axis([-1 2 -1.5 1.25])
grid on
```



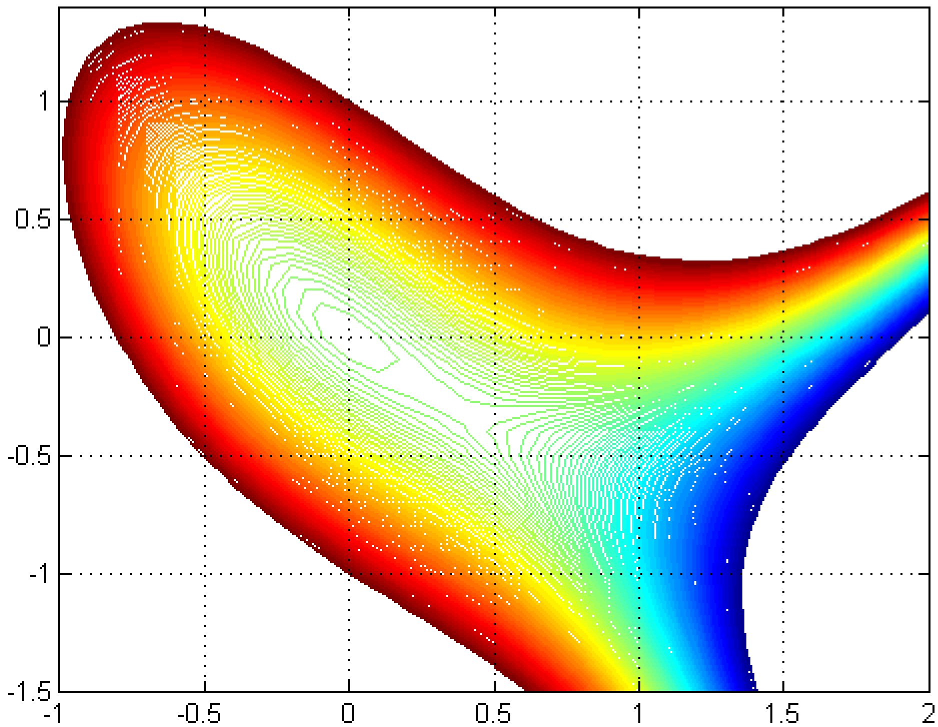

```
b=0.6;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1.5 1.25])

grid on
```



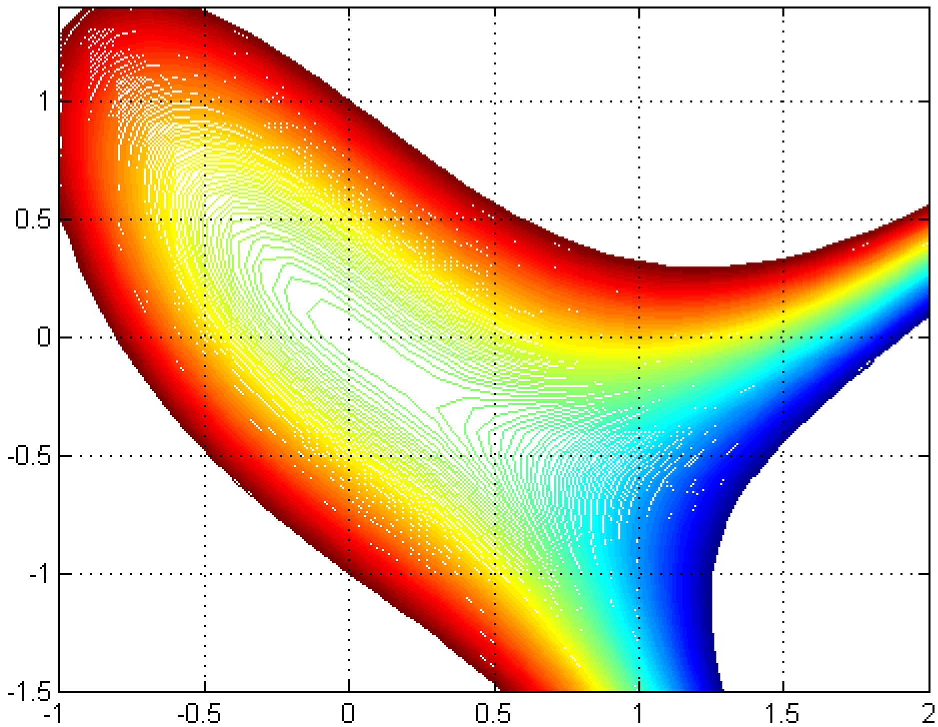

```
b=0.7;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1.5 1.25])

grid on
```



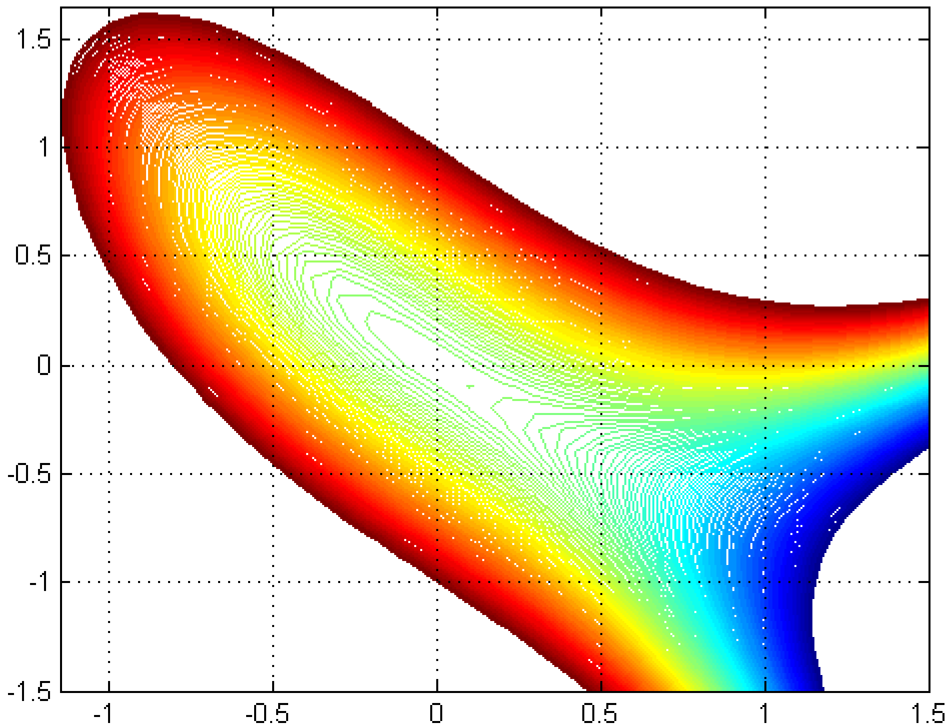

```
b=0.8;

f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;

x=-15:.1:15; y=-15:.1:15;

[X,Y]=meshgrid(x,y);

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-1 2 -1.5 1.4])

grid on
```




```
b=0.9;
f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;
x=-15:.1:15; y=-15:.1:15;
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
level=-.5:.005:.5;
contour(X,Y,Z,level)
axis([-1 2 -1.5 1.4])
grid on
```




```
b=1.0;
f=@(x,y)(y.^2/2)+(b*y.*x)+(x.^2)/2 -(x.^3)/3;
x=-15:.1:15; y=-15:.1:15;
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
level=-.5:.005:.5;
contour(X,Y,Z,level)
axis([-1.15 1.5 -1.5 1.65])
grid on
```


1.5
1
0.5
0
-0.5
-1
-1.5
-1
-0.5
0
0.5
1
1.5

Kevin Pasinski Math 246 0231 Extra Credit

given: $\frac{d^2x}{dt^2} + b\frac{dx}{dt} + x - x^2 = 0$ where $b = [0,1]$

$\frac{dx}{dt} = y$

$\frac{dy}{dt} = -by + x^2 - x$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{dy}{dx} = \frac{-by+x^2-x}{y} \rightarrow y\,dy = (-by+x^2-x)\,dx$ integrate both sides

$\rightarrow \frac{y^2}{2} = -byx + \frac{1}{3}x^3 - \frac{1}{2}x^2 \rightarrow \frac{1}{2}y^2 + byx - \frac{1}{3}x^3 + \frac{1}{2}x^2 = 0$

$\rightarrow H(x,y) = \frac{1}{2}y^2 + byx - \frac{1}{3}x^3 + \frac{1}{2}x^2 = 0$

$\frac{dx}{dt} = y = f(x,y)$

$\frac{dy}{dt} = -by + x^2 - x = g(x,y)$

Stationary Points $= (0,0)$ and $(1,0)$

$\frac{dx}{dt} = y = 0 \rightarrow y = 0$

$\frac{dy}{dt} = -by + x^2 - x = 0$

$= -b(0) + x^2 - x = 0$

$= x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

Linearization

$A = \begin{pmatrix} d_x f & d_y f \\ d_x g & d_y g \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2x-1 & -b \end{pmatrix}$

@ $(0,0)$ $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\det(A-\lambda I) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \det\begin{pmatrix} -\lambda & 1 \\ -1 & -\lambda \end{pmatrix}$

$= \lambda^2 + 1$ $\lambda^2 = -1$ $\lambda = \pm i$

$(A-\lambda I) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix}$ (Clockwise)

@ $(1,0)$ $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\det(A-\lambda I) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \det\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}$

$= \lambda^2 - 1$ $\lambda^2 = 1$

$\lambda = \pm 1$

for $\lambda = 1$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

$-x_1 + x_2 = 0$ $-x_1 = -x_2$ $x_1 = x_2$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

for $\lambda = -1$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $x_1 + x_2 = 0$ $x_1 = -x_2$ $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

So when $b = 0$, there is a stationary point at $(0,0)$ which is a center which is stable. Also there is a stationary point at $(1,0)$ which is a saddle which is unstable.