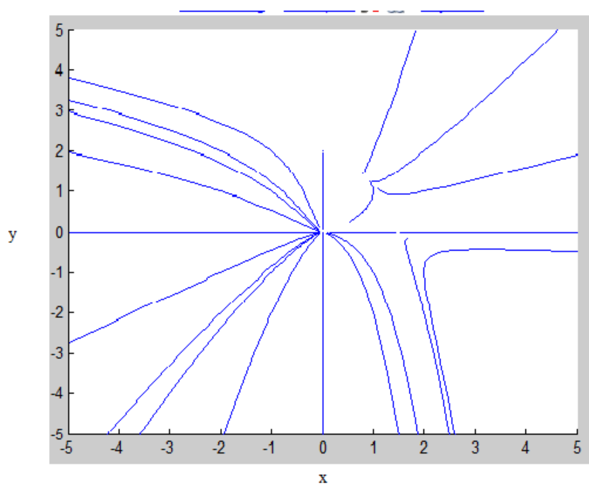
James Polivka
Math246
5/11/2009

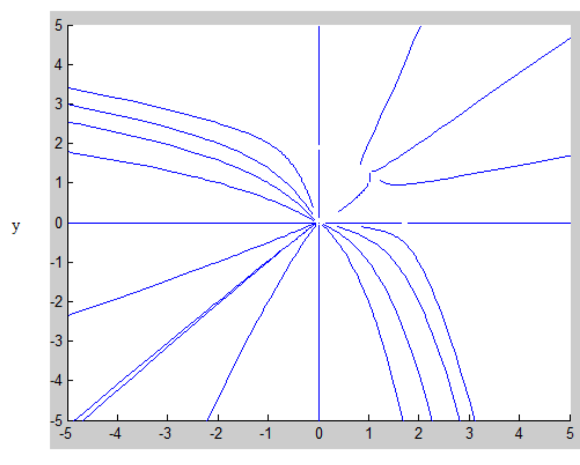
A Tale of Two Derivatives

In this project, I show the progression from section 9.4 problem 1 in the textbook to section 9.5 problem 2 using a parameter alpha at intervals of 0.2. Alpha was chosen so that when $\alpha = 1$, problem 9.4.1 is the only graph shown, and when $\alpha = 0$, the graph appears only as the second system of differential equations. The change progresses gradually, but there are several points at which critical points change from one type to another in order to reach a final product. To save space, I have written the assigned equation on the last page of the assignment.



Problem 9.4.1

This is a phase plane for problem 1 in section 9.4. The critical points are the origin; (0,2); (1.5,0); and (.8,1.4). At the origin, there is an unstable node. (0,2) is an unstable saddle point, as is point (1.5,0). The final point is an asymptotically stable node.



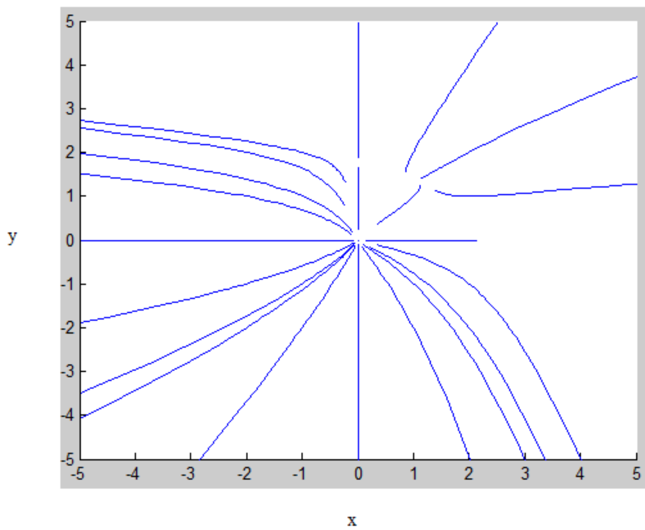
This graph shows the system of equations when the parameter $\alpha = .2$. The critical points of this particular system can be given through MATLAB code

```
>> syms x y
>> sys1 = .8*x*(1.5-x-.5*y)+.2*x*(1-.5*y);
```

```
>> sys2 = .8*y*(2-y-.75*x)+.2*y*(-.25+.5*x);
>> [xc, yc] = solve(sys1, sys2, x, y);
>> disp('Critical Points:'); disp([xc yc])
Critical Points:

[     0,     0]
[   7/4,     0]
[     0, 31/16]
[ 23/26, 18/13]
```

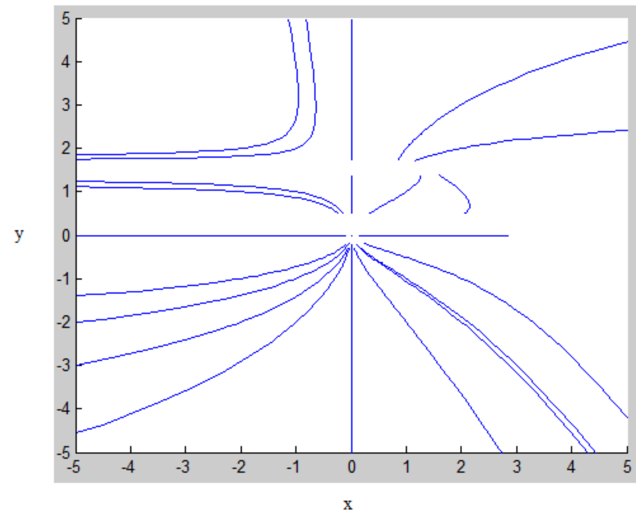
to be the origin; (1.75,0); (0,1.9375); and (.8846,1.3846). Although it is somewhat difficult to make these points out on the graph, one can see holes where the critical points on the origin lie, and nodes at the other two.



This graph is of the parameter α = .4. Using the same MATLAB code as in the previous graph, the critical points are found to be

```
[     0,     0]
[  13/6,     0]
[     0,  11/6]
[ 46/47, 67/47].
```

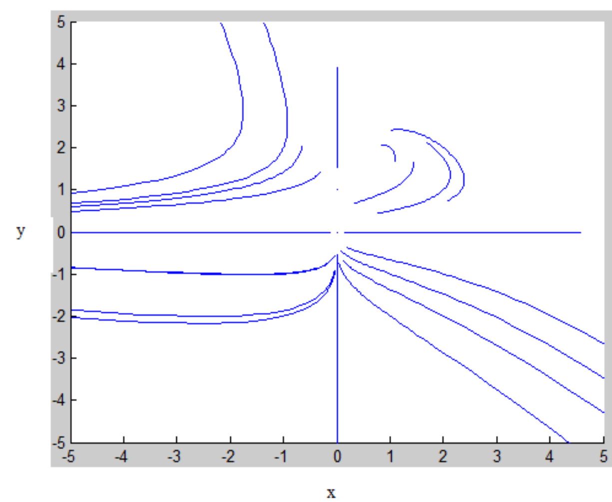
One can still observe the points on the origin and axes. The point in the first quadrant is moving slightly with each variation of the parameter.



This system, with the parameter α = .6, still has 4 critical points, which are found with MATLAB to be

[0, 0]
[3, 0]
[0, 13/8]
[31/32, 13/8].

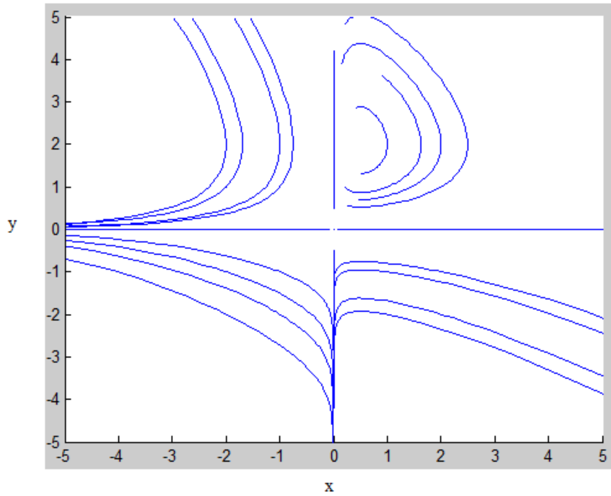
As one can see, the point on the origin and the point in the first quadrant are unstable nodes still. The points on the axes are moving away from the origin as the parameter increases.



The critical points of this graph with a parameter of α = .8 are

[0, 0]
[0, 1]
[11/2, 0]
[8/11, 21/11].

The point at the origin still shows properties of a node but is starting to look like a saddle. The first quadrant point is starting to resemble a center point, which is what the point eventually turns into. The axis points are still visible on the graph.



Problem 9.5.2

In this final picture, the parameter has progressed completely from the first exercise to the second; from zero to 1. Now there are only two critical points, found by MATLAB to be

[0, 0]

and

[1/2, 2].

The origin is an unstable saddle point, and the second point is an indeterminate spiral point. The other two points have disappeared due to the relative simplicity of the second part of the equation to the first.

Assigned Equation:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \end{pmatrix} = (1-\alpha)\cdot\begin{pmatrix} x(1.5-x-.5y) \\ y(2-y-.75x) \end{pmatrix} + \alpha\cdot\begin{pmatrix} x(1-.5y) \\ y(-.25+.5x) \end{pmatrix}$$

Evaluate the parameter α from 0 to 1.