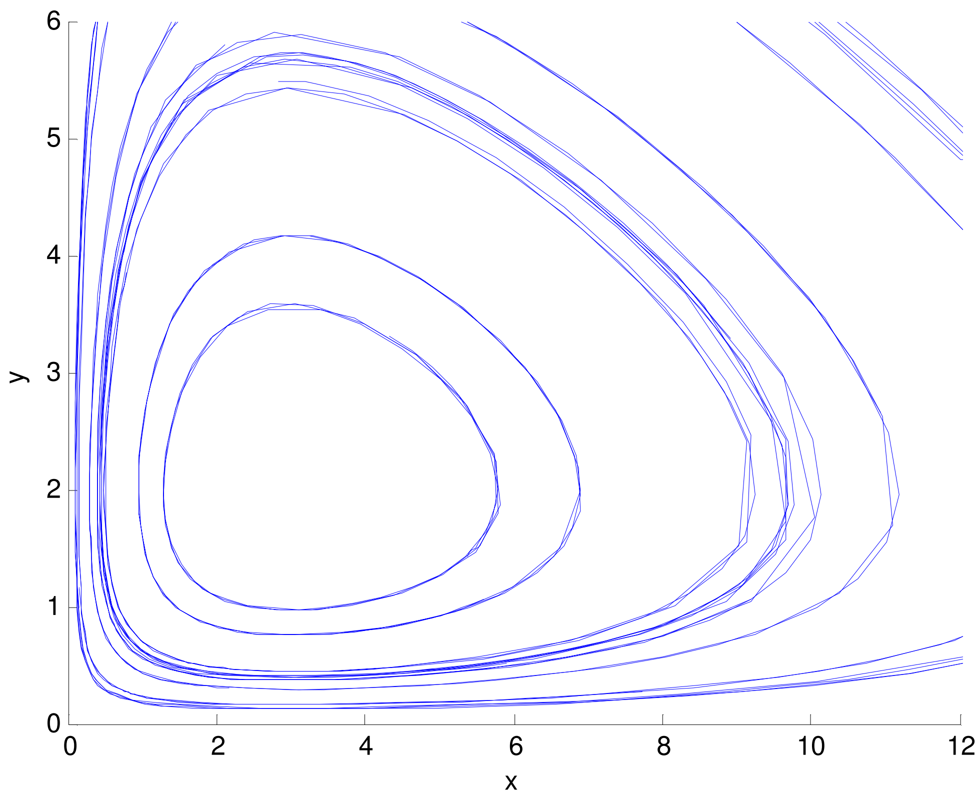
Michael Schwartz

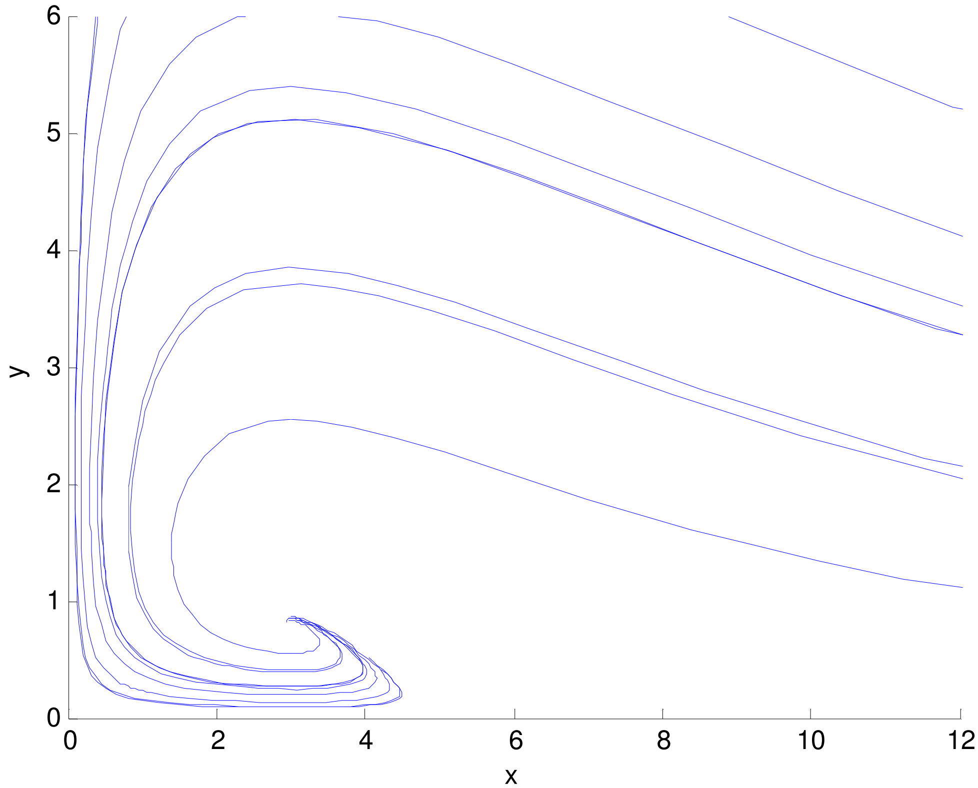
Math 246 Extra Credit Predator-Prey Model

Competitive level of predators amongst themselves: 0.8
Competitive level of prey amongst themselves: 2.4

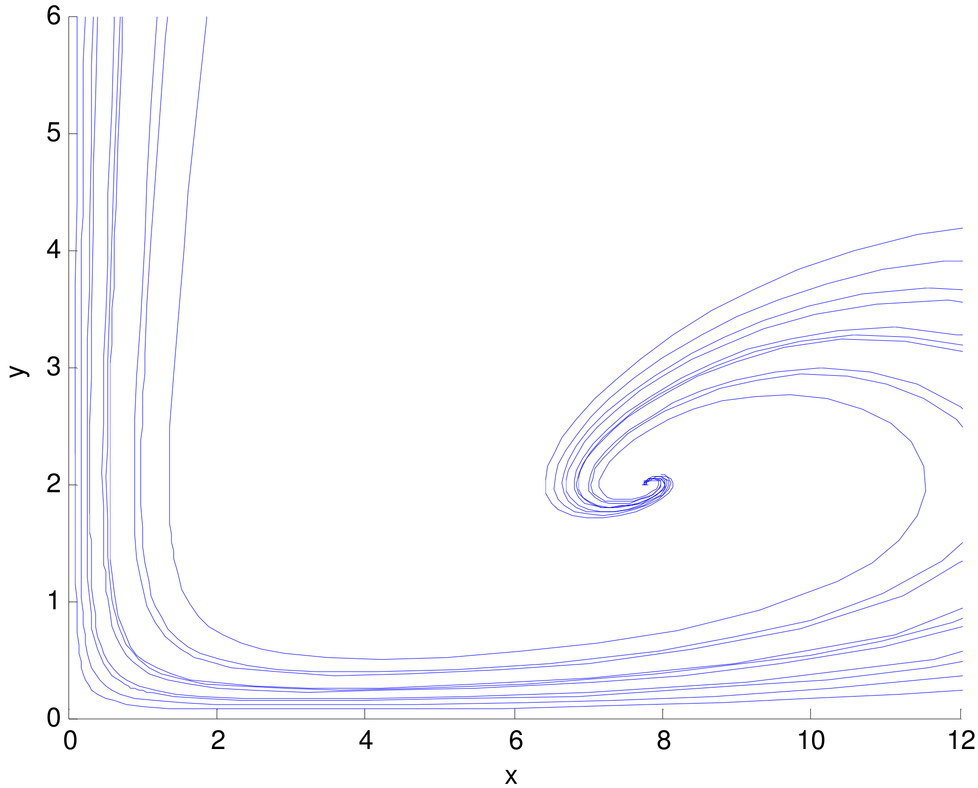
Neither the Predators and the Prey compete within their own respective species for limited food.
This will give the phase plane of dx/dt=x(4-2y), dy/dt=y(-3+x)
Critical Points:
[0, 0]
[3, 2]



Only the Predators compete within their own species for limited food.
This will give the phase plane of dx/dt=x(4-2y-zeta*x), dy/dt=y(-3+x)
Critical Points:
[0, 0]
[5, 0]
[3, 4/5]



Only the Prey compete within their own species for limited food.
This will give the phase plane of dx/dt=x(4-2y), dy/dt=y(-3+x-delta*y)
Critical Points:
[0,   0]
[0, -5/4]
[39/5,   2]



Both the Predators and the Prey compete within their own respective species for limited food.

This will give the phase plane of dx/dt=x(4-2y-zeta*x), dy/dt=y(-3+x-delta*y)

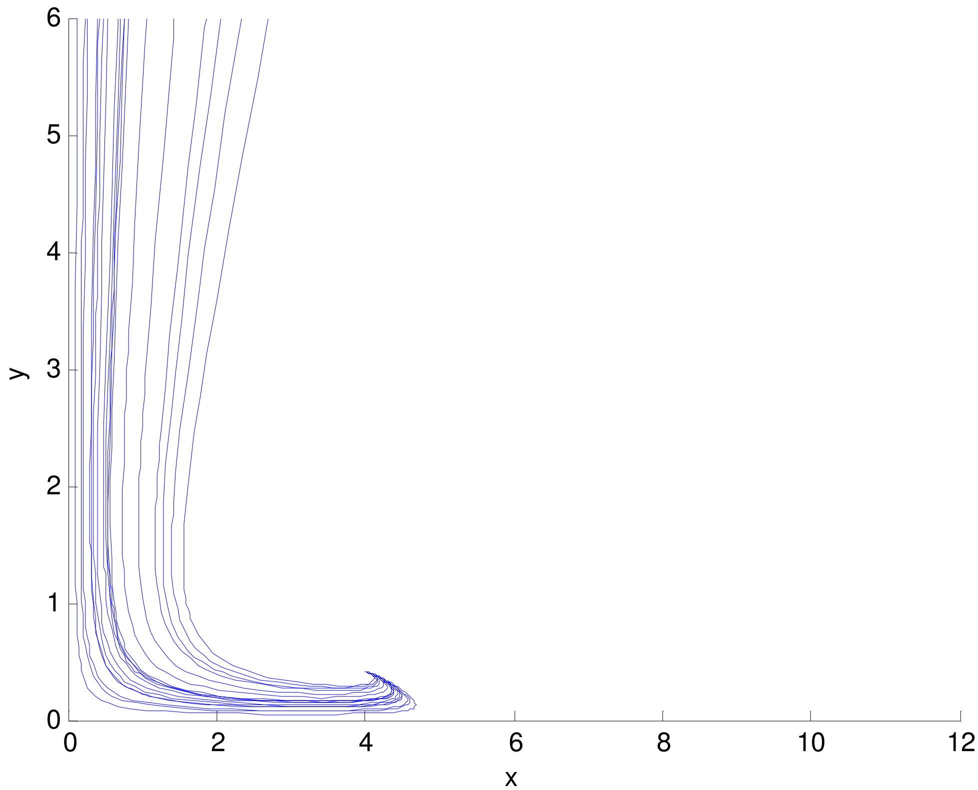
Critical Points:

[0, 0]

[0, -5/4]

[5, 0]

[195/49, 20/49]



Each of these graphs give separate stories when viewed independently and together. It can be noted that in Figure 2, with the competitive predators, that the predator population falls dangerously low due to competition, but then recovers. On the other hand, in Figure 3, with the competitive prey, both populations fall dangerously low, then rise high again, and repeats this pattern, spiraling closer to the critical point each cycle. What I will demonstrate now is where the transition point is where Figure 2 transforms into Figure 3, in terms of delta and zeta.

Through experimentation, I have found that higher values of zeta "straighten" the graph away from the semi-elliptical shape. Lower values of zeta will cause a mild spiral. Low values of delta "break up" the elliptical continuity, and cause the graph to spiral, whereas higher values of delta will tend to make the graph "hug" the axes in the first quadrant. I have furthermore found that increasing the value of zeta will keep the x-component of the critical point at 3, but decreases the value of the y-component. I have also found that, on the other hand, increasing the value of the delta component will retain the y-component value of the critical point at 2, but increases the value of the x-component.

In essence, the transition point from when Figure 2 transforms into Figure 3 is when both zeta and delta are zero, from which the graphs will move in separate directions. In addition, the graph where zeta is between 0.5-1 is similar in shape to the graph where delta is greater than 2. When delta is 2, there seems to be a singularity where MatLab cannot draw the phase plane. At values a little greater than 2, MatLab can draw the phase plane eventually, after long delays.

From my analysis, the transition point from when Figure 2 transforms into Figure 3 is where zeta=0.8, and delta=2.4. This yields the population dynamics:

$$dx/dt = x(4-2y-0.8x)$$

$$dy/dt=y(-3+x-2.4y)$$

where x is the population of the predators and y is the population of the prey. This population dynamic is the best estimate for the transition point for when the model simulating only the predators being competitive for food is similar in shape to the model of only the prey being competitive for food.

```
%% Michael Schwartz
%A Study of Predator and Prey Interactions
%Dr. C.D. Levermore
%Math 246, Section 0232

clear all
close all
warning off all
clc

options = odeset('RelTol',1e-3,'AbsTol',1e-3);
syms x y;

zeta = input('Competitive level of predators amongst themselves: ');
delta = input('Competitive level of prey amongst themselves: ');

% Non-Competitive Predators and Prey
disp('Neither the Predators and the Prey compete within their own respective species for limited food.')
disp('This will give the phase plane of dx/dt=x(4-2y), dy/dt=y(-3+x)')

sys1 = x*(4-2*y);
sys2 = y*(-3+x);
[xc yc] = solve(sys1,sys2,x,y);
disp('Critical Points:');
disp([xc yc])

figure(1);

hold on
f = @(t,x) [(x(1))*(4-2*x(2)); ...
   (x(2))*(-3+x(1))];
for a = [1 2]
   for b = [0.1:0.2:0.9]
      [t,xa]=ode45(f,[0 3], [a*b a*2.5*(1-b)], options);
      plot(xa(:,1), xa(:,2))
      [t,xa]=ode45(f,[0 -3], [a*b a*2.5*(1-b)], options);
      plot(xa(:,1), xa(:,2))
   end
end
axis([0 12 0 6])
xlabel('x')
ylabel('y')
hold off

%% Competitive Predators
```

```
disp('Only the Predators compete within their own species for limited food.')
disp('This will give the phase plane of dx/dt=x(4-2y-zeta*x), dy/dt=y(-3+x)')

sys1 = x*(4-zeta*x-2*y);
sys2 = y*(-3+x);
[xc yc] = solve(sys1,sys2,x,y);
disp('Critical Points:');
disp([xc yc])

figure(2);

hold on
f = @(t,x) [(x(1))*(4-zeta*x(1)-2*x(2)); ...
    (x(2))*(-3+x(1))];
for a = [1 2]
    for b = [-0.9:0.2:0.9]
        [t,xa]=ode45(f,[0 3], [a*b a*2.5*(1-b)], options);
        plot(xa(:,1), xa(:,2))
        [t,xa]=ode45(f,[0 -3], [a*b a*2.5*(1-b)], options);
        plot(xa(:,1), xa(:,2))
    end
end
axis([0 12 0 6])
xlabel('x')
ylabel('y')
hold off

%% Competitive Prey
disp('Only the Prey compete within their own species for limited food.')
disp('This will give the phase plane of dx/dt=x(4-2y), dy/dt=y(-3+x-delta*y)')

sys1 = x*(4-2*y);
sys2 = y*(-3+x-delta*y);
[xc yc] = solve(sys1,sys2,x,y);
disp('Critical Points:');
disp([xc yc])

figure(3);

hold on
f = @(t,x) [(x(1))*(4-2*x(2)); ...
    (x(2))*(-3+x(1)-delta*x(2))];
for a = [1 2]
    for b = [-0.9:0.2:0.9]
        [t,xa]=ode45(f,[0 3], [a*b a*2.5*(1-b)], options);
        plot(xa(:,1), xa(:,2))
```

```
        [t,xa]=ode45(f,[0 -3], [a*b a*2.5*(1-b)], options);
        plot(xa(:,1), xa(:,2))
    end
end
axis([0 12 0 6])
xlabel('x')
ylabel('y')
hold off

%% Competitive Predators and Prey
disp('Both the Predators and the Prey compete within their own respective species for limited food.')
disp('This will give the phase plane of dx/dt=x(4-2y-zeta*x), dy/dt=y(-3+x-delta*y)')

sys1 = x*(4-zeta*x-2*y);
sys2 = y*(-3+x-delta*y);
[xc yc] = solve(sys1,sys2,x,y);
disp('Critical Points:');
disp([xc yc])

figure(4);

hold on
f = @(t,x) [(x(1))*(4-zeta*x(1)-2*x(2)); ...
    (x(2))*(-3+x(1)-delta*x(2))];
for a = [1 2]
    for b = [-0.9:0.1:0.9]
        [t,xa]=ode45(f,[0 3], [a*b a*2.5*(1-b)], options);
        plot(xa(:,1), xa(:,2))
        [t,xa]=ode45(f,[0 -3], [a*b a*2.5*(1-b)], options);
        plot(xa(:,1), xa(:,2))
    end
end
axis([0 12 0 6])
xlabel('x')
ylabel('y')
hold off
```