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When Competition Turns Nasty...

We combine the models 9.4 #1 and 9.5 #1, by adding the right-hand side of both equations. The sum is weighted by Alfa, in such a way that if the Competing Species model (9.4) has a weight of Alfa, then the Predator-Prey model (9.5) has a weight of (1-Alfa). The Alfa coefficient is then modified on a scale 0 to 1. As a result, the Competing Species model gradually morphs into Predator-Prey model. This document analysis behavior of this system as Alfa is increased by 0.2 intervals. It is interesting to observe how the behavior around the stable critical point changes from asymptotic node, to twist, to spiral, and finally center. Here you will find Phase Portraits for all 6 systems, as well as all direction fields, as well as critical values and eigenvectors. Varying values of eigenvectors are particularly instructive, and can be easily observed through appropriate hyperlinks. The document also presents the code that was used to produce charts, as well as to find critical points and eigenvalues. While you can find more detailed information about Competing Species Model in Chapter 9.4, and about Predator-Prey Model in chapter 9.5, this document is meant primarily for illustrative purposes. Enjoy.

Competing Species Model (Problem 9.4 #1)

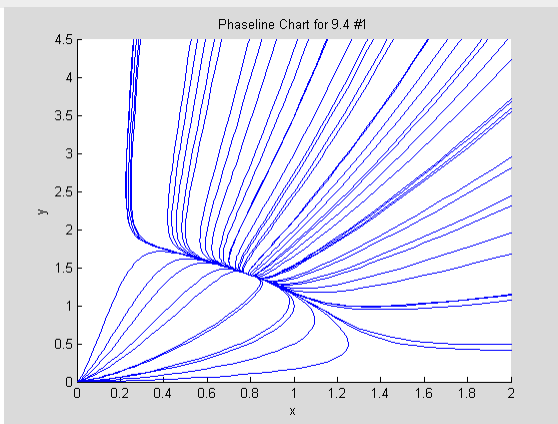
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```
close all
clear all
warning off all
```

```

figure; hold on
f = @(t,x) [x(1)*(1.5-x(1)-0.5* x(2));...
           x(2)*(2-x(2)-0.75*x(1))];
for a = 0.25:0.25:1.75
    for b = 0.5:0.5:4
        [t,xa] = ode45(f, [0 10], [a,b]);
        plot(xa(:,1), xa(:,2))
        [t,xa] = ode45(f, [0 -5], [a,b]);
        plot(xa(:,1), xa(:,2))
        title(['Phaseline Chart for 9.4 #1'])
        xlabel x
        ylabel y
    end
end
axis ([0,2,0,4.5])

```



Predator-Prey Model (Problem 9.5 #1)

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```

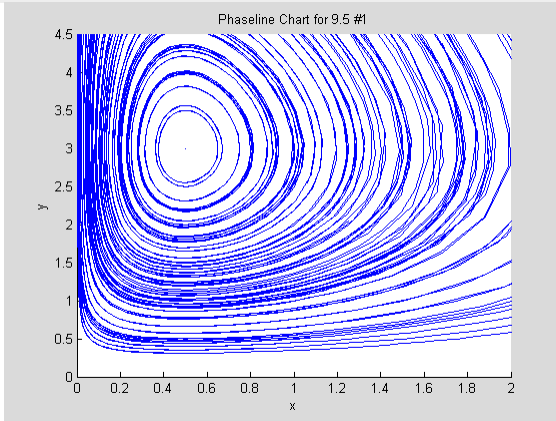
close all
clear all
% This is Problem 9.5 #1
warning off all
figure; hold on
f = @(t,x) [x(1)*(1.5-0.5* x(2));...
           x(2)*(-0.5+x(1))];
for a = 0.25:0.25:1.75
    for b = 0.5:0.5:4
        [t,xa] = ode45(f, [0 10], [a,b]);
        plot(xa(:,1), xa(:,2))
        [t,xa] = ode45(f, [0 -5], [a,b]);
        plot(xa(:,1), xa(:,2))
        title(['Phaseline Chart for 9.5 #1'])
    end
end

```

```

        xlabel x
        ylabel y
    end
end
axis ([0,2,0,4.5])

```



Phase Portraits for the Combined Model

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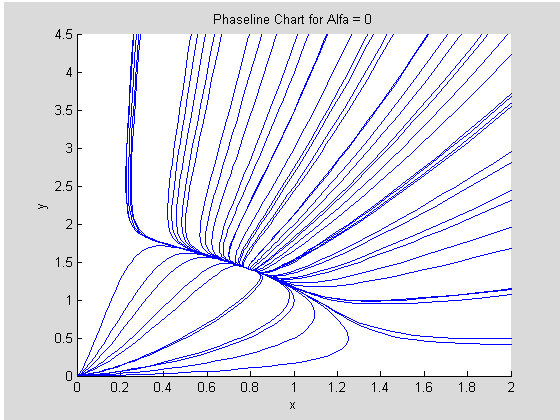
```

close all
clear all
warning off all

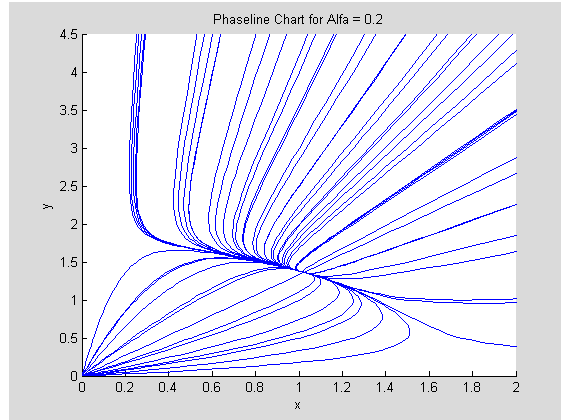
for a = 0:0.20:1
    f = @(t,x) [(1-a)*(x(1)*(1.5-x(1)-0.5*x(2)))+a*(x(1)*(1.5-0.5*
x(2)));...
                (1-a)*(x(2)*(2-x(2)-0.75*x(1)))+a*(x(2)*(-0.5+x(1)))]);
    figure; hold on
    for c = 0.25:0.25:1.75
        for d = 0.5:0.5:4
            [t,xa] = ode45(f, [0 10], [c,d]);
            plot(xa(:,1), xa(:,2))
            [t,xa] = ode45(f, [0 -5], [c,d]);
            plot(xa(:,1), xa(:,2))
            title(['Phase line Chart for Alfa = ' num2str(a)])
            xlabel x
            ylabel y
        end
    end
    axis ([0,2,0,4.5])
end

```

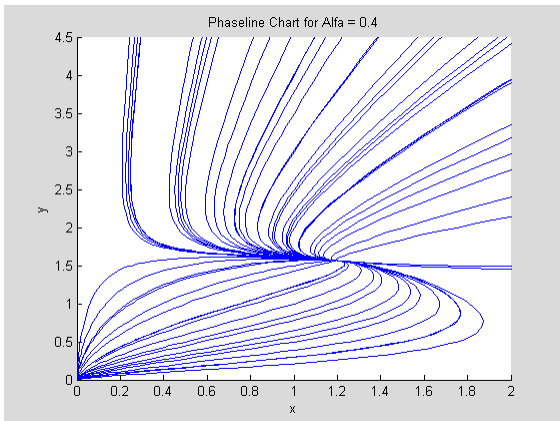
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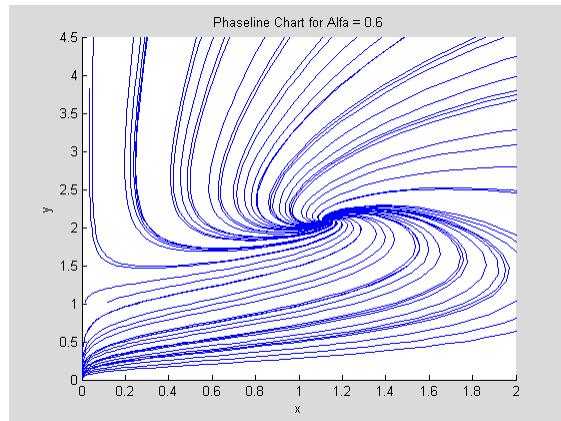
[Direction Field for Alfa = 0](#)
[View Critical Points and Eigenvalues for Alfa = 0](#)



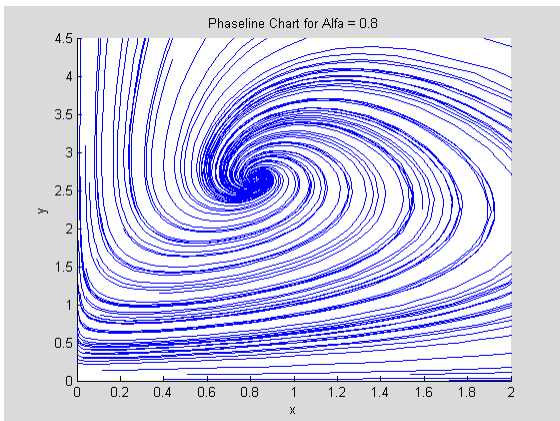
[Direction Field for Alfa = 0.2](#)
[View Critical Points and Eigenvalues for Alfa = 0.2](#)



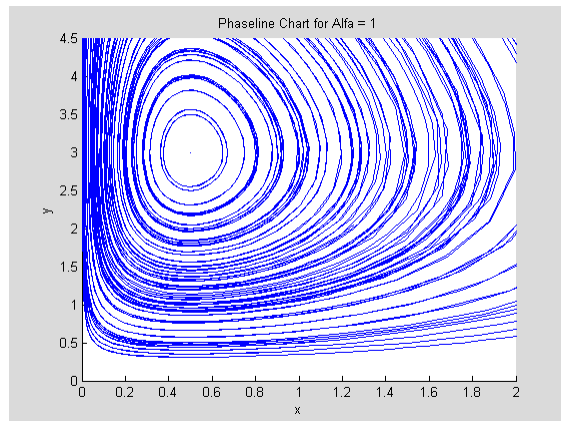
[Direction Field for Alfa = 0.4](#)
[View Critical Points and Eigenvalues for Alfa = 0.4](#)



[Direction Field for Alfa = 0.6](#)
[View Critical Points and Eigenvalues for Alfa = 0.6](#)



[Direction Field for Alfa = 0.8](#)
[View Critical Points and Eigenvalues for Alfa = 0.8](#)



[Direction Field for Alfa = 1](#)
[View Critical Points and Eigenvalues for Alfa = 1](#)

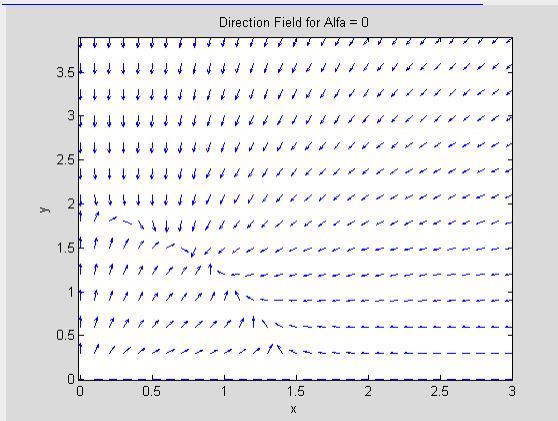
Direction Field for the Combined Model

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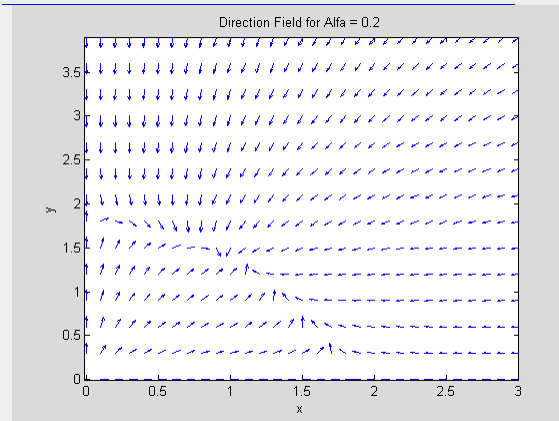
```
clear all
%syms a y x
for a = 0:0.2:1
    figure
    [X, Y] = meshgrid(0:0.1:3, 0:0.3:4);
    U = (1-a)*(X.*(1.5-X-0.5*Y))+a*(X.*(1.5-0.5* Y));
    V = (1-a)*(Y.*(2-Y-0.75*X))+a*(Y.*(-0.5+X));
    L = sqrt((U/2).^2+(V/4.5).^2);
    quiver(X, Y, U./L, V./L, 0.4);
    axis tight
    title (['Direction Field for Alfa = ' num2str(a)])
    xlabel x
    ylabel y
end
```

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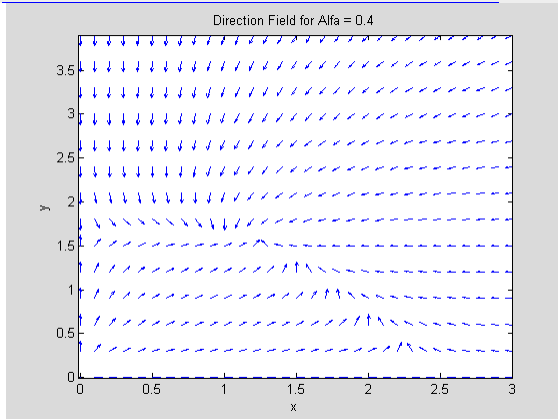
[Back to Phaseline for Alfa = 0](#)



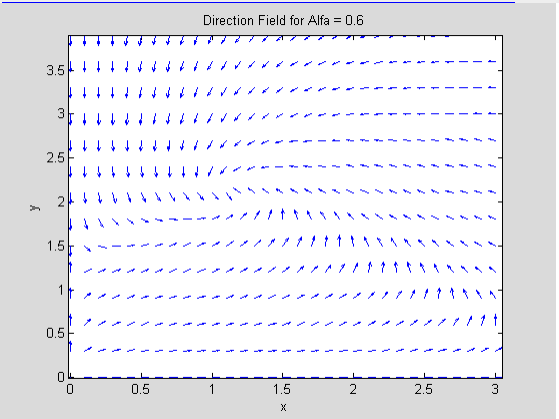
[Back to Phaseline for Alfa = 0.2](#)



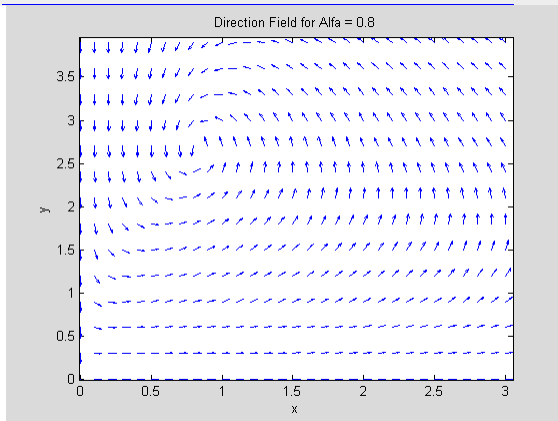
[Back to Phaseline for Alfa = 0.4](#)



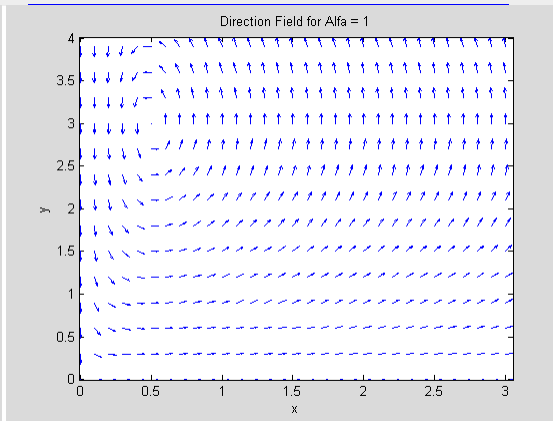
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[Back to Phaseline for Alfa = 0.8](#)



[Back to Phaseline for Alfa = 1](#)



Critical Points and Eigenvalues for the Combined Model

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```
syms x y a
for a = 0:0.20:1
    sys1 = (1-a)*(x*(1.5-x-0.5*y))+a*(x*(1.5-0.5* y));
    sys2 = (1-a)*(y*(2-y-0.75*x))+a*(y*(-0.5+x));
    disp(['THE FOLLOWING IS FOR THE VALUE OF ALFA '])
    a
    [xc, yc] = solve(sys1, sys2, x, y);
    disp('Critical points:'); disp([xc yc])
    A = jacobian([sys1 sys2], [x y]);
    evals = eig(A);
    s = numel([xc]);
    for n = 1:s
        disp(['Eigenvalues at the following stationary point'])
        x_ = xc(n,1)
        y_ = yc(n,1)
        disp(['are displayed below'])
        disp(double(subs(evals, {x,y}, {x_,y_})))
    end
end
end
```

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[Back to Phaseline for Alfa = 0](#)

THE FOLLOWING IS FOR THE VALUE OF ALFA

a =

0

Critical points:

[0, 0]

[0, 2]

[3/2, 0]

[4/5, 7/5]

Eigenvalues at the following stationary point

x_ =

0

y_ =

0

are displayed below

1.5000

2.0000

Eigenvalues at the following stationary point

x_ =

0

y_ =

2

are displayed below

-2.0000

0.5000

Eigenvalues at the following stationary point

x_ =

3/2

y_ =

0

are displayed below

-1.5000

0.8750

Eigenvalues at the following stationary point

$x_1 =$

$4/5$

$y_1 =$

$7/5$

are displayed below

-1.8141

-0.3859

[Back to Phaseline for Alfa = 0.2](#)

THE FOLLOWING IS FOR THE VALUE OF ALFA

a =

0.2000

Critical points:

[0, 0]

[15/8, 0]

[0, 15/8]

[45/44, 15/11]

Eigenvalues at the following stationary point

x_ =

0

y_ =

0

are displayed below

1.5000

1.5000

Eigenvalues at the following stationary point

x_ =

15/8

y_ =

0

are displayed below

-1.5000

0.7500

Eigenvalues at the following stationary point

x_ =

0

y_ =

15/8

are displayed below

-1.5000
0.5625

Eigenvalues at the following stationary point

$x_1 =$

45/44

$y_1 =$

15/11

are displayed below

-1.5000
-0.4091

[Back to Phaseline for Alfa = 0.4](#)

THE FOLLOWING IS FOR THE VALUE OF ALFA

a =

0.4000

Critical points:

[0, 0]

[5/2, 0]

[0, 5/3]

[80/67, 105/67]

Eigenvalues at the following stationary point

x_ =

0

y_ =

0

are displayed below

1.0000

1.5000

Eigenvalues at the following stationary point

x_ =

5/2

y_ =

0

are displayed below

-1.5000

0.8750

Eigenvalues at the following stationary point

x_ =

0

y_ =

5/3

are displayed below

-1.0000
0.6667

Eigenvalues at the following stationary point

$x_ =$

80/67

$y_ =$

105/67

are displayed below

-1.0719
-0.5848

[Back to Phaseline for Alfa = 0.6](#)

THE FOLLOWING IS FOR THE VALUE OF ALFA

$a =$

0.6000

Critical points:

[0, 0]
[0, 5/4]
[15/4, 0]
[35/31, 65/31]

Eigenvalues at the following stationary point

$x_ =$

0

$y_ =$

0

are displayed below

0.5000
1.5000

Eigenvalues at the following stationary point

$x_ =$

0

$y_ =$

5/4

are displayed below

-0.5000

0.8750

Eigenvalues at the following stationary point

$x_1 =$

15/4

$y_1 =$

0

are displayed below

-1.5000

1.6250

Eigenvalues at the following stationary point

$x_2 =$

35/31

$y_2 =$

65/31

are displayed below

-0.6452 - 0.5636i

-0.6452 + 0.5636i

[Back to Phaseline for Alfa = 0.8](#)

THE FOLLOWING IS FOR THE VALUE OF ALFA

a =

0.8000

Critical points:

[0, 0]

[15/2, 0]

[60/73, 195/73]

Eigenvalues at the following stationary point

x_ =

0

y_ =

0

are displayed below

0
1.5000

Eigenvalues at the following stationary point

x_ =

15/2

y_ =

0

are displayed below

-1.5000
4.8750

Eigenvalues at the following stationary point

x_ =

60/73

y_ =

195/73

are displayed below

-0.3493 - 0.8242i

-0.3493 + 0.8242i

[Back to Phaseline for Alfa = 1](#)

THE FOLLOWING IS FOR THE VALUE OF ALFA

a =

1

Critical points:

[0, 0]

[1/2, 3]

Eigenvalues at the following stationary point

x_ =

0

y_ =

0

are displayed below

-0.5000

1.5000

Eigenvalues at the following stationary point

x_ =

1/2

y_ =

3

are displayed below

0 - 0.8660i

0 + 0.8660i