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% Justin Shearer
% Math 246 Section 0222
% Extra Credit Problem

% d^2x/dt^2 + b(dx/dt)^3 + sin(x) = 0
% d/ (x) = (        y       )  where (x) = (  x  )
% dt (y) = (-by^3 - sin(x))        (y) = (dx/dt)

% h(x, y) = y^2 - 2*xy^3 - cos(x)

b = 1.000;
% I varied the values of b from 0 to 1 at different intervals in order to
% produce the different pictures.
[X1 Y1] = meshgrid(-15:.01:15);
hold on
contour(X1, Y1, Y1.^2 - 2.*b.*X1.*Y1.^3 - 2.*cos(X1))
[x1 y1] = meshgrid(-15:1:15);
U = y1;
V = -b.*y1^3 - sin(x1);
L = sqrt((U).^2 + (V).^2);
quiver(x1, y1, U./L, V./L, 0.4);
axis tight
xlabel x
ylabel y
title 'Phase Diagram for d^2x/dt^2 + b(dx/dt)^3 + sin(x) = 0 when b = 1.000'
hold off
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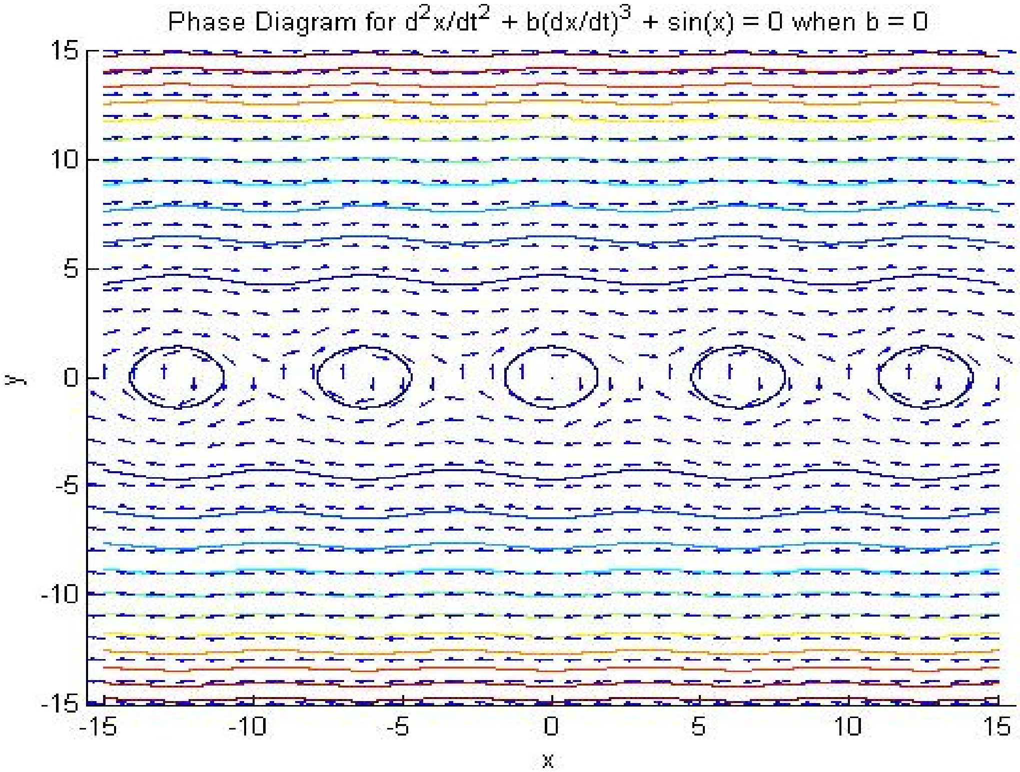
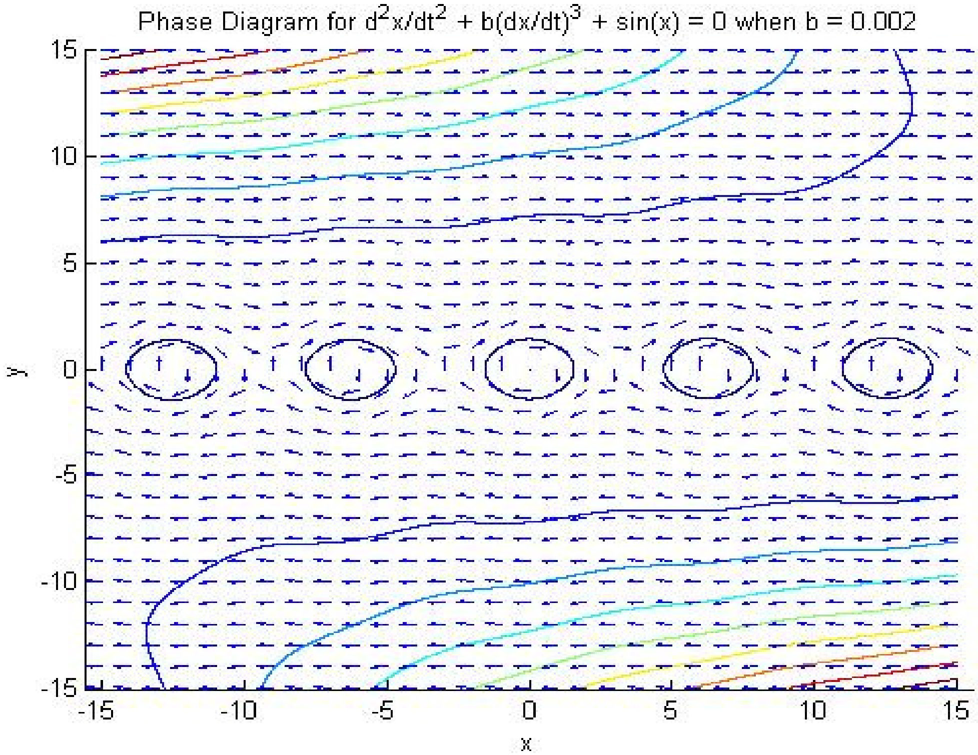
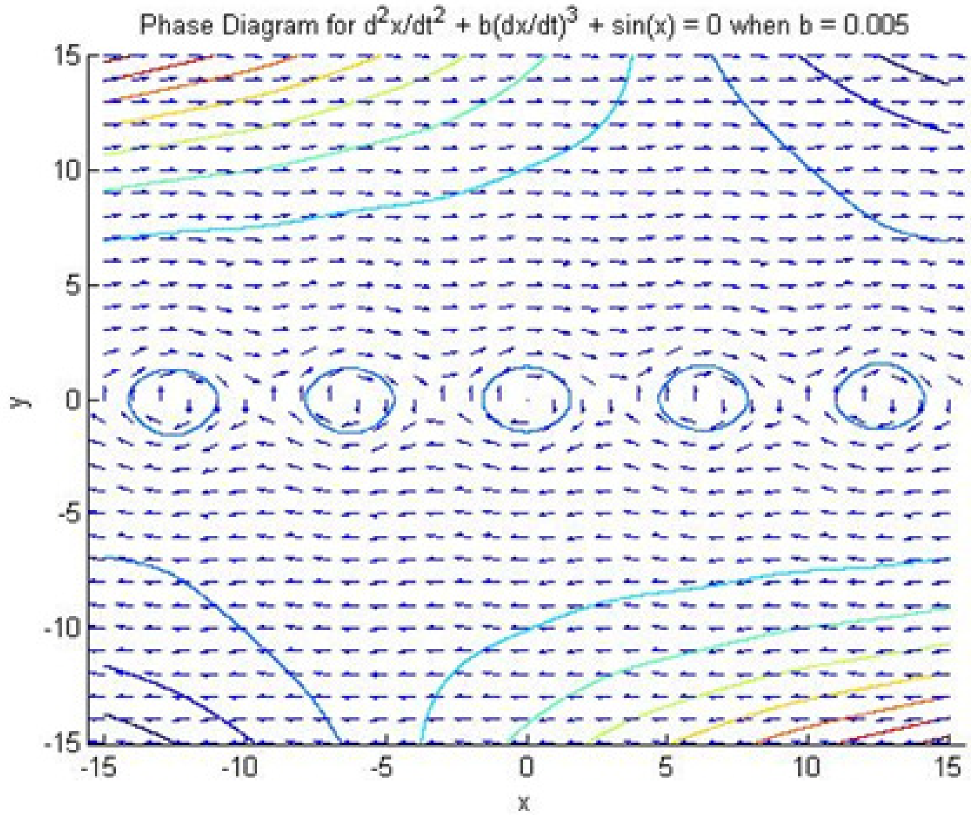


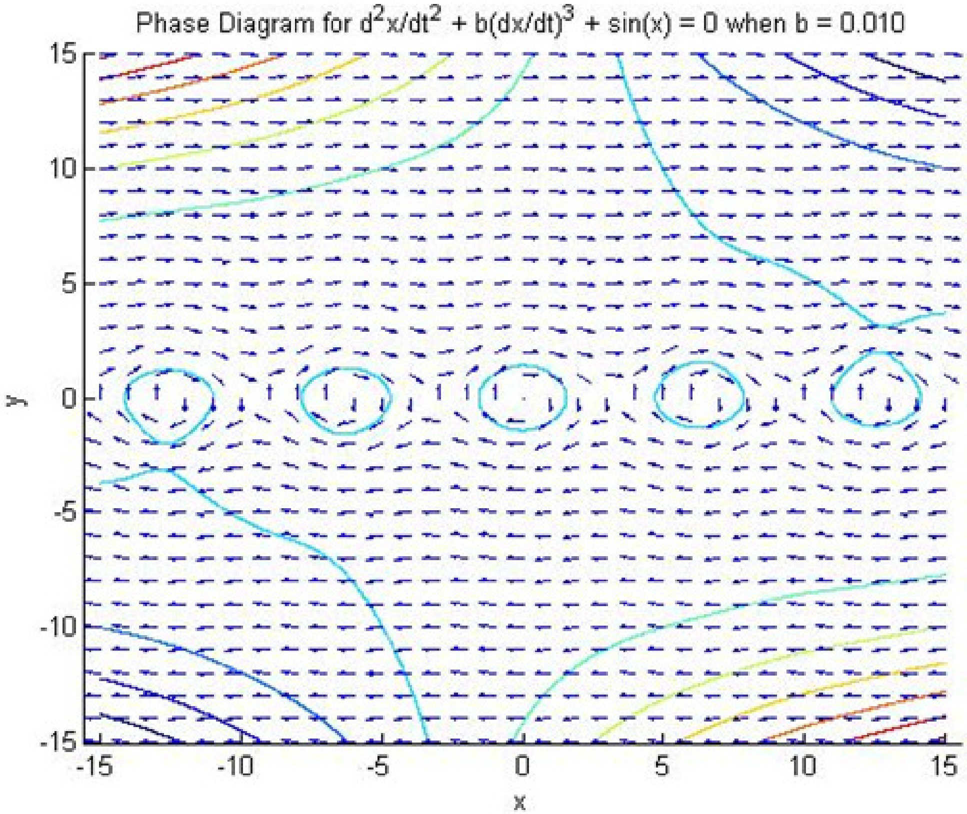
When b = 0, the pendulum is undamped and it just osculates between a number and the opposite of that number (i.e. 5 and -5). There is no other force acting on the pendulum so it just swings back and forth from one value to the other without slowing down or stopping. The points along the y axis surrounded by circles are the critical points where the pendulums are stationary.



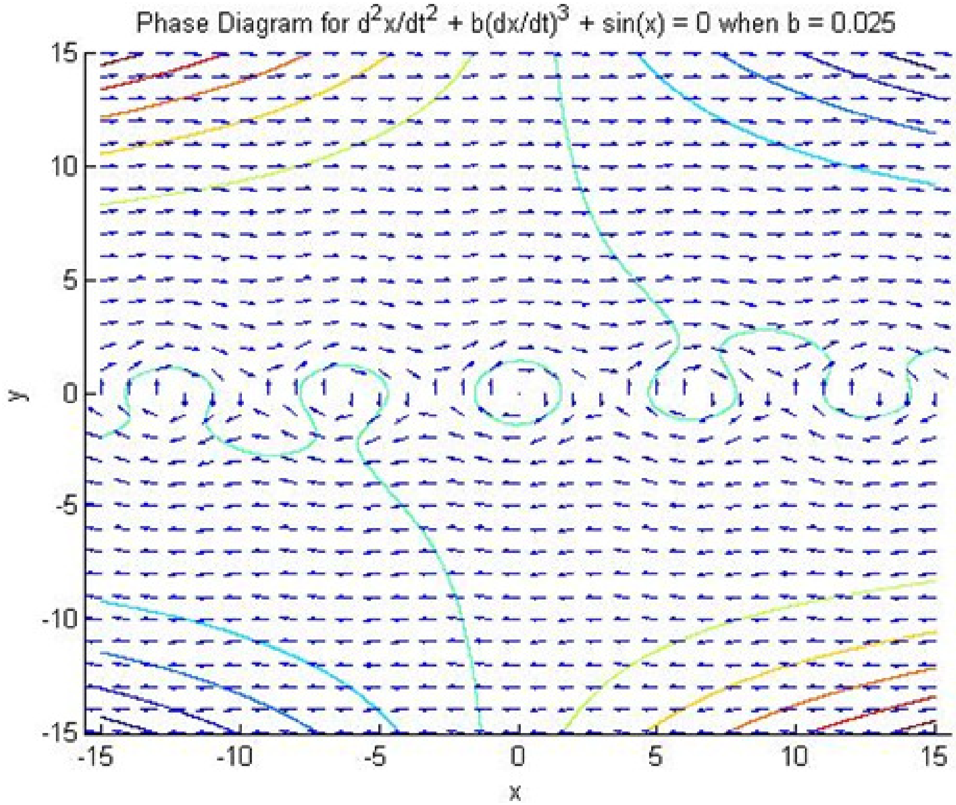
When b = .002, the dampening can be seen acting on it a little. The oscillations that were previously parallel to one another have started to bend back away from each other at opposite sides while the other sides start bending towards one another. This shows that now the pendulum no longer continues on forever but does eventually start to approach zero. The side that bends up appears as if it might be causing the pendulum to swing a little to the side as well.



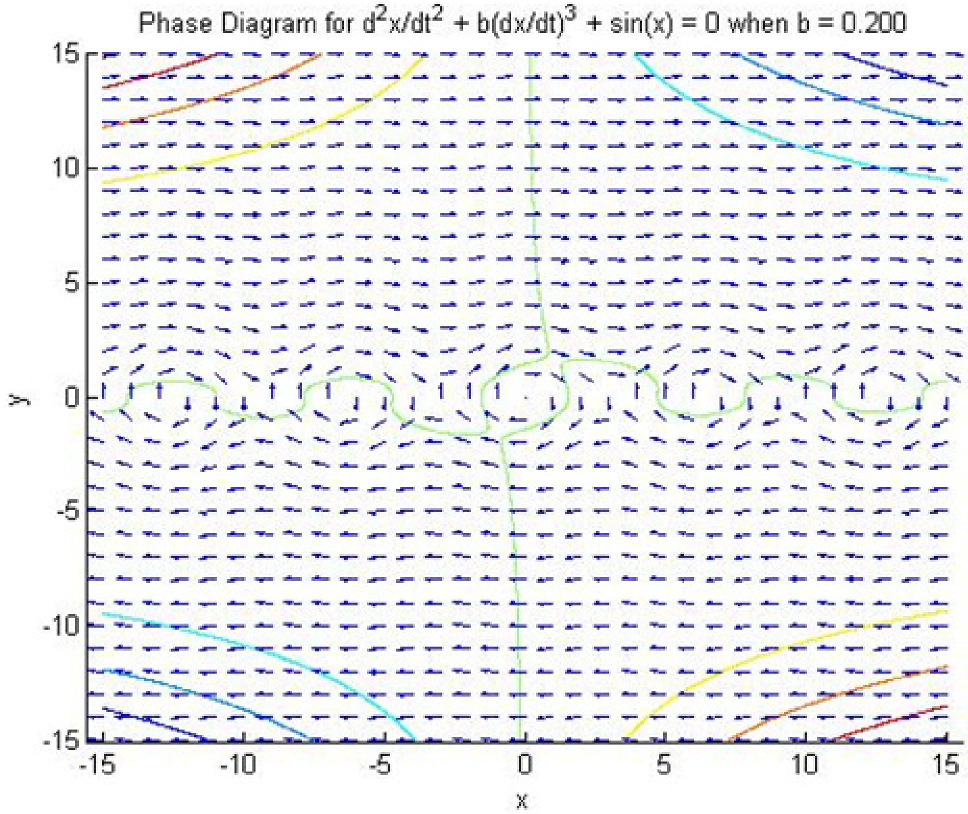
When b = .005, the phase portrait starts produce trajectories that seem to bend in towards the center. This makes it appear as if the dampening is starting to swing from side to side as well as backward and forward. This makes it seem as though the pendulum is oscillating back and forth and circling the origin as it does this.



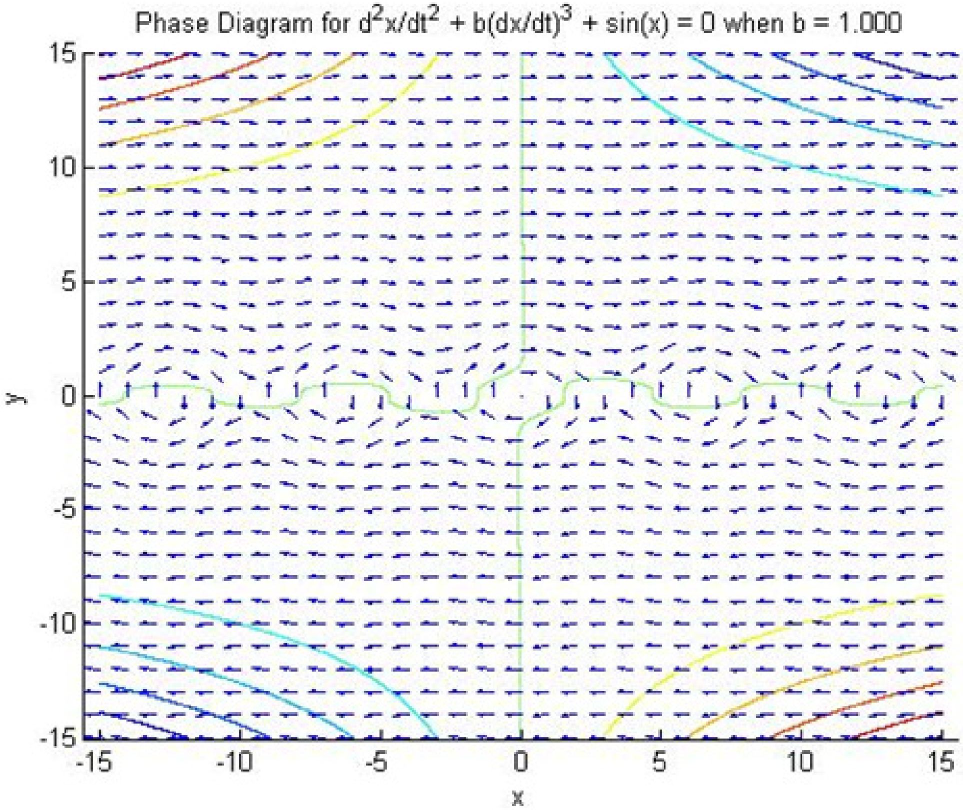
When b = .010, one of the trajectories on either side starts pulling at the center and sources in the middle of the picture. This shows that the continued dampening is now starting to deform the area in the center where the pendulum is in equilibrium. This could either mean that the positions when it is both at the top and the bottom are now a little deformed and either more likely to fall out of place (at the top) or fall into place (at the bottom).



When b = .025, the same thing is happening again but with the second equilibrium points, closer to the center of the phase diagram. The trajectory has attached to the equilibrium point and started to pull it apart into something completely different. It begins to look as if the pendulum comes into a kind of asymptotic region where it goes back in line with the center and then spins off again. It is doing a similar pattern as before where it was spinning around the center but now it is also osculating in a more continuous line with the center until it gets close to the center when it is pushed away again.



When b = .200, the trajectory that was close to the center finally touches the center equilibrium point. The pendulum is doing the same kind of thing here as it was in the previous picture but in a more periodic way. When it has a lot of energy it is still osculating back and forth and spinning around the center but it is losing energy as well. When it loses enough to get close to the center I starts to go back and forth at a more exact rate but there now seems to be an asymptote at the center of the pendulum. This could in a way symbolize that it will take the pendulum a long time to finally stop.



When b = 1, the picture is just a more refined version of when b was equal to .200. The same trajectories are shown in the corners displaying how it is osculating and spinning around the center. The trajectory that connected to the middle of the picture has pulled the center apart even more and is now turning into more of what looks like an asymptote. The pendulum appears to be doing the same thing as in the previous picture. When it loses energy from the dampening it just begins to go back and forth almost looking like the undamped pendulum.