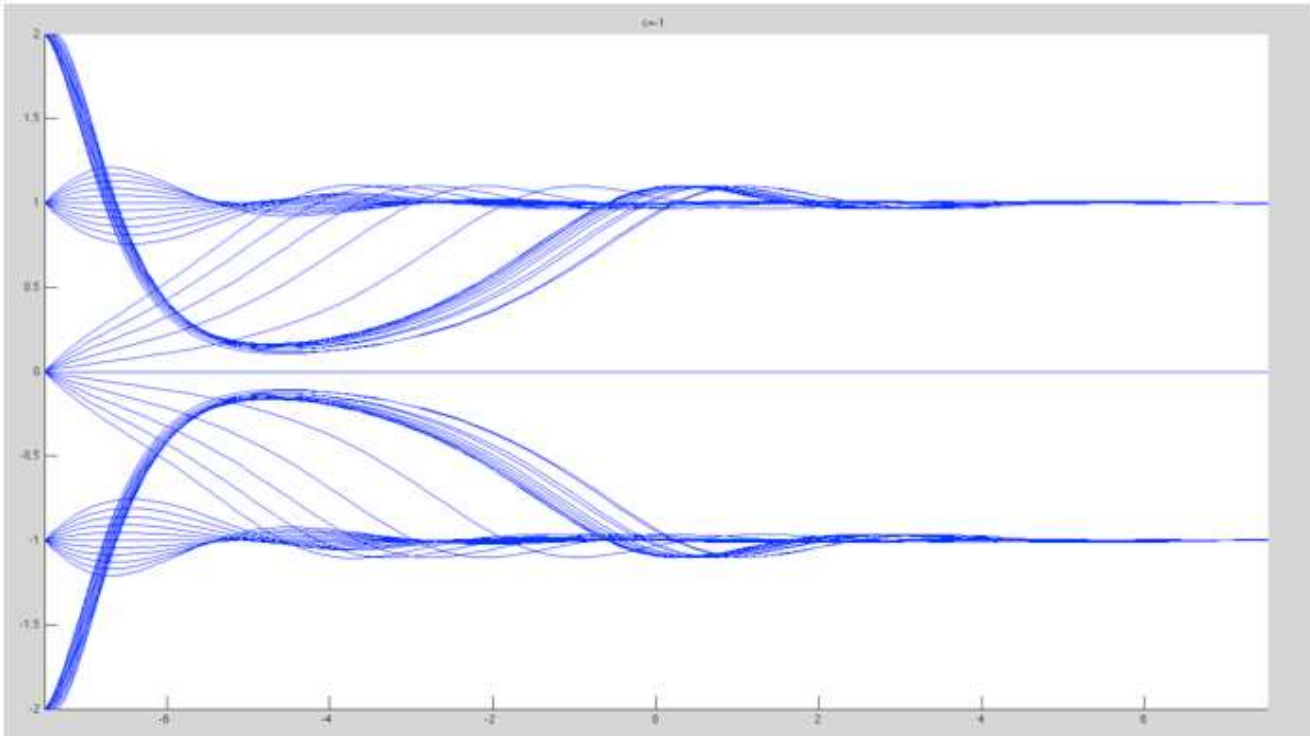


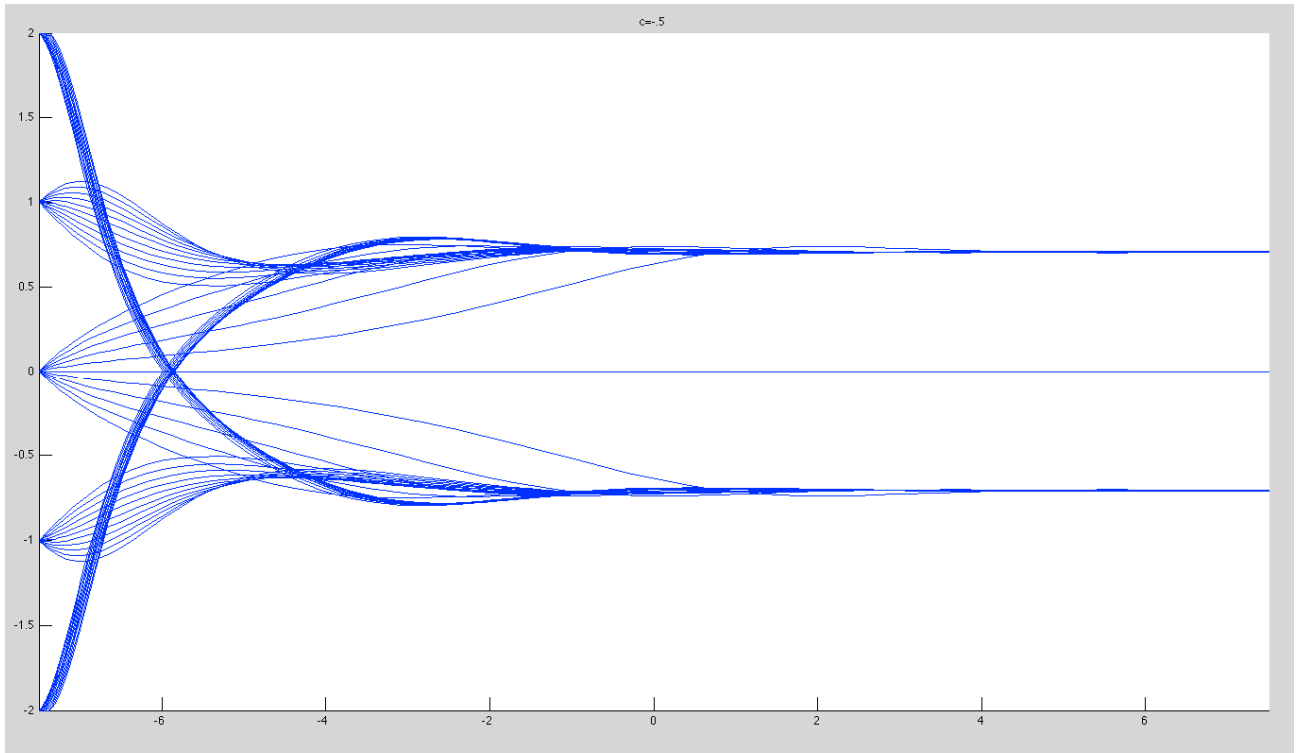
Initial Value Problem:

$$Dx^2/Dt^2 + dx/dt + x^3 + c*x = 0, \text{ for } c = -1 \text{ to } 1$$

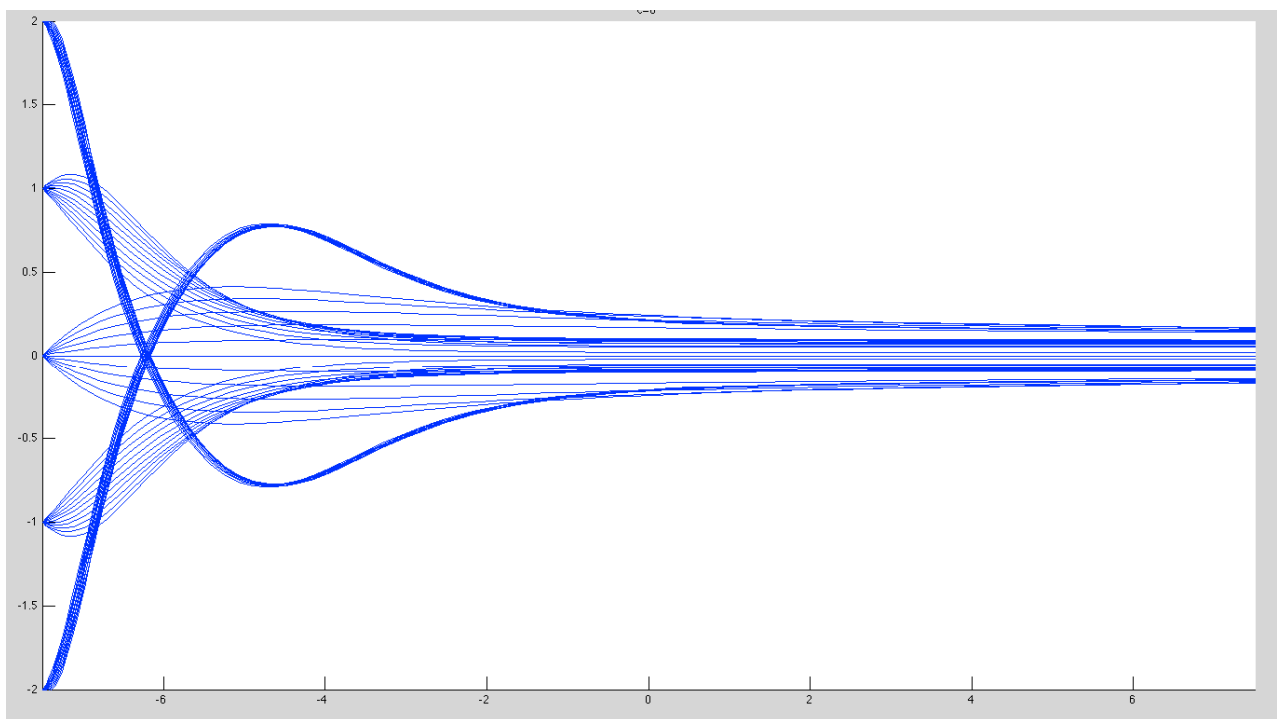
Starting at $c=-1$ the plot from -7.5 to 7.5 starts with curves that originate from $y = -2$ and 2 that converge to -1 and 1 , respectively. All lines that originate from 0 converge towards -1 and 1 except for one that remains at $y=0$.



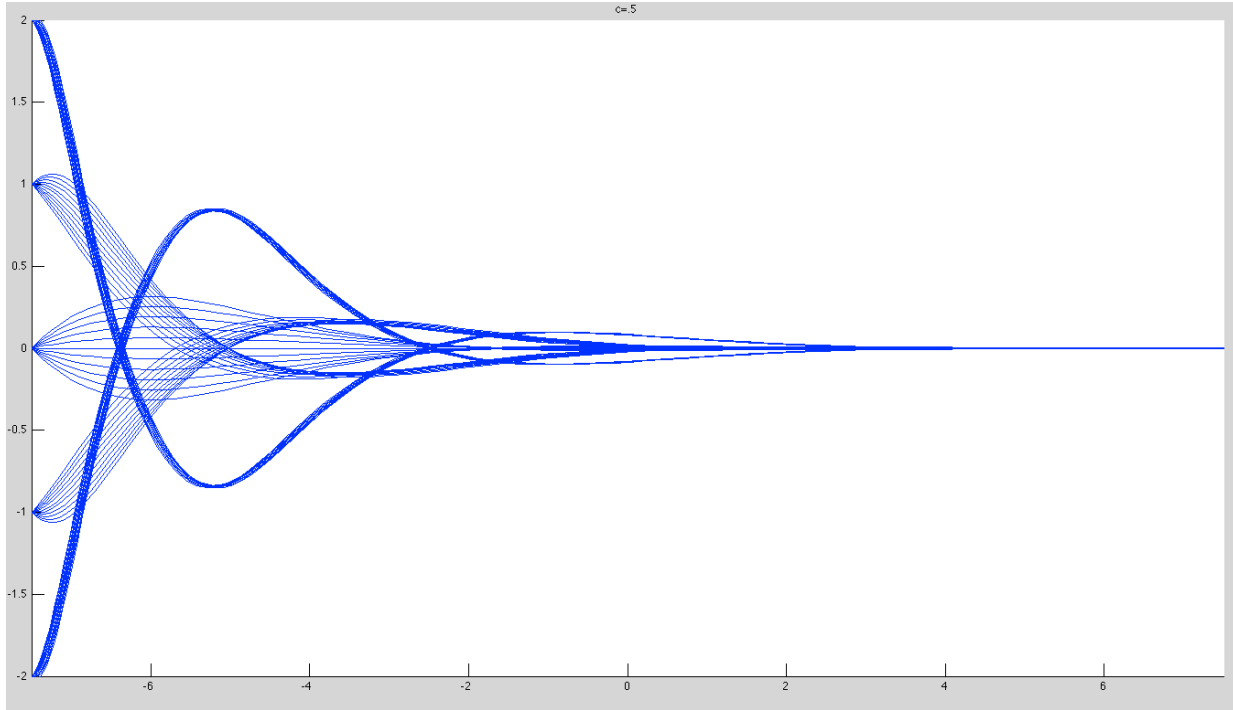
When c becomes -0.5 the lines originating from -2 and 2 switch and converge to 1 and -1 respectively and are “pulling in” the rest of the graph towards the center.



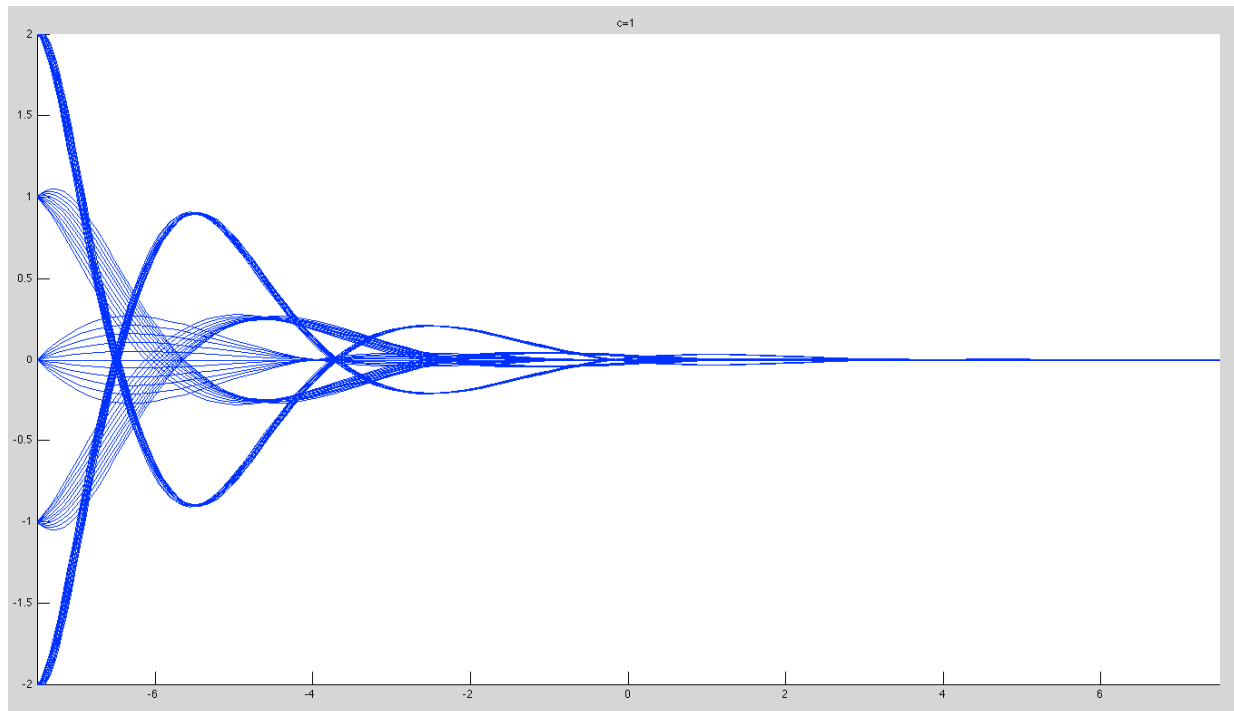
When c becomes 0, a more dramatic change occurs. All lines start to converge towards $y=0$. The curves originating from -2 and 2 bend towards the center creating a formation similar to node-antinode structure.



When $c=.5$, the curves have started to collapse on $y=0$. There are nodes at $(-6.37,0)$ and $(-2.43,0)$ $(-.509,0)$ $(-.23,0)$ $(2.71,0)$



At $c=1$, the curves have fully collapsed on $y=0$ at more negative values of x . This has caused more oscillation about $y=0$ as well, leading to six nodes: -6.48 -3.77 $-.117$ -5.09 -2.43 0 2.5



When the graphs are observed at higher and lower ranges the pattern of nodes and anti nodes continues, only at higher and lower amplitudes. If the graphs were zoomed in at higher positive values, you would see infinitely small nodes decreasing and vice versa for increasing negative values on the x-axis.

Code:

```

%%Dx^2/Dt^2 + dx/dt+x^3 + c*x = 0 for c= -1, 1

clear
syms c

figure
c=1;
rhs = @(t,x) [x(2); -x(2) - x(1)^3 - x(1)*c];

[xa, ya] = ode45(rhs, [-7.5 7.5], [50 55]);

hold on
for x0 =-2:2
    for xp0 = -.5:.1:.5
        [tfor, xfor] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
hold off
title 'c=1'
axis([-7.5 7.5 -2 2])

figure
c=0;
rhs = @(t,x) [x(2); -x(2) - x(1)^3-x(1)*c];

[xa, ya] = ode45(rhs, [-7.5 7.5], [50 55]);

hold on
for x0 =-2:2
    for xp0 = -.5:.1:.5
        [tfor, xfor] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
title 'c=0'
hold off
axis([-7.5 7.5 -2 2])

figure

```

```

c=-1;
rhs = @(t,x) [x(2); -x(2) - x(1)^3-x(1)*c];

[xa, ya] = ode45(rhs, [-7.5 7.5], [50 55]);

hold on
for x0 =-2:2
    for xp0 = -.5:.1:.5
        [tfor, xfor] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
title 'c=-1'
hold off
axis([-7.5 7.5 -2 2])

figure
syms c
c=.5;
rhs = @(t,x) [x(2); -x(2) - x(1)^3 - x(1)*c];

[xa, ya] = ode45(rhs, [-7.5 7.5], [50 55]);

hold on
for x0 =-2:2
    for xp0 = -.5:.1:.5
        [tfor, xfor] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
title 'c=.5'
axis([-7.5 7.5 -2 2])

figure
syms c
c=-.5;
rhs = @(t,x) [x(2); -x(2) - x(1)^3 - x(1)*c];

[xa, ya] = ode45(rhs, [-7.5 7.5], [50 55]);

hold on
for x0 =-2:2
    for xp0 = -.5:.1:.5
        [tfor, xfor] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        [tbak, xbak] = ode45(rhs, [-7.5 7.5], [x0 xp0]);
        plot(tfor, xfor(:,1))
        plot(tbak, xbak(:,1))
    end
end
end

```

```
hold off
title 'c=-.5'
axis([-7.5 7.5 -2 2])
```