Kevin Smith – Matlab Second Order Equation.

%%The following phase portraits depict the behavior of the second order %%differential equation (D2x/D2t)+b(dx/dt)^3-x^2+x=0. The sequence of %%pictures are a result of changing the variable "b" from 0-1, and even %%beyond. The behavior of the fucntion changes drastically immediately %%after "b" becomes non-zero. The stability of the origin becomes %%immediately affected however it remains stable. The saddle point at (1,0) that %%originally existed when b=0 has now drastically changed to create an %%almost inverted version of the previous picture. As "b" goes to infinity %%these points become increasingly less stable until nearly becoming %%unstable. One can see this in the last picture where "b" is increased to %%5 and then 100. The stationary point (1,0) has now been taken over %%and (0,0) has now taken the shape of an unstable saddle point. However, %%it is interesting to note that even though these points appear to become %%unstable, the linearization (as shown in the handwritten %%derivations) shows that even as "b" becomes non negative the matrices of %%the stationary points stay the same because "y" is still 0. Therefore, %%the stationary points (0,0) and (1,0) should remain as a stable center and %%an unstable saddle point. The shape of the graph outside of the stationary points drastically changes after "b" becomes non-zero because of the y dependent cubic function that now exists. The tangent lines which the level curves approach as "t" decreases are no longer linear because the derivative of the function "y^3"

becomes "y^2." This is clearly seen as the tangent lines are parabolic arcs rather than straight lines when "b" is non-zero.

Kevin Smith

$$\frac{dx^2}{dt^2} + b\left(\frac{dx}{dt}\right)^3 - x^2 + x = 0$$

$$f(x,y) = \frac{dx}{dt} = y$$

$$g(x,y) = \frac{dy}{dt} = -b\left(\frac{dx}{dt}\right)^3 + x^2 - x = -b\left(y^3\right) + x^2 - x$$

Level Curves

$$\frac{dy}{dx} = \frac{-by^3 + x^2 - x}{y}$$

$$H(x,y) = \frac{y^2}{2} + by^3x + \frac{x^2}{2} - \frac{x^3}{3}$$

stationary Points

$$0 = y$$

$$0 = b(0)^3 + x^2 - x$$

$$x(1-x) = 0$$

Points $= (0,0)$, $(1,0)$

Linearization

$$\begin{pmatrix} d_x f & d_y f \\ d_x g & d_y g \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2x-1 & -3y^2(b) \end{pmatrix}$$

at stationary pint

$(0,0)$

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$p(z) = z^2 + 1$

$\pm i$

Center

Stable

$(1,0)$

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$p(z) = z^2 - 1$

roots $(z+1)(z-1)$

$\lambda = -1, 1$

saddle, unstable

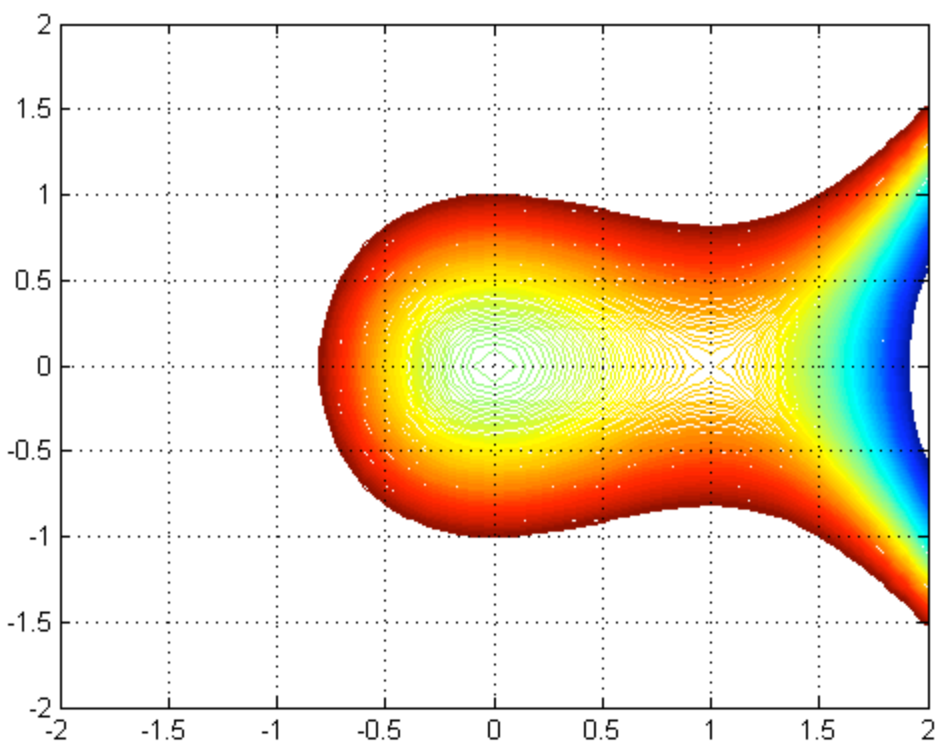
repelling

eigen vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

```
b=0

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.5:.005:.5;

contour(X,Y,Z,level)

axis([-2 2 -2 2])

grid on
```

b =

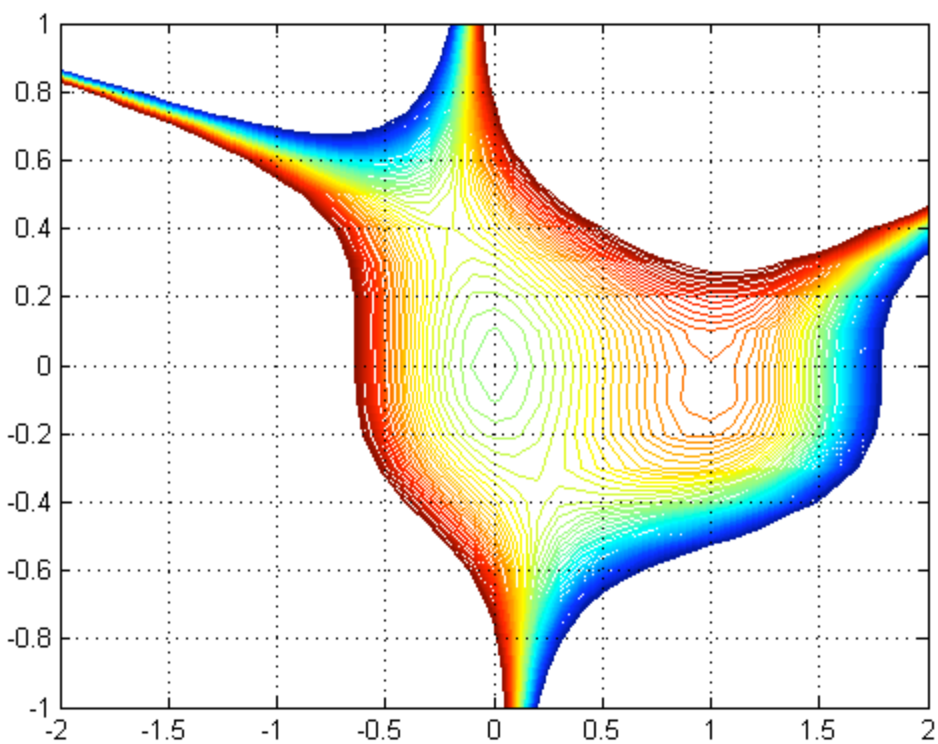
0



```
b=.1

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

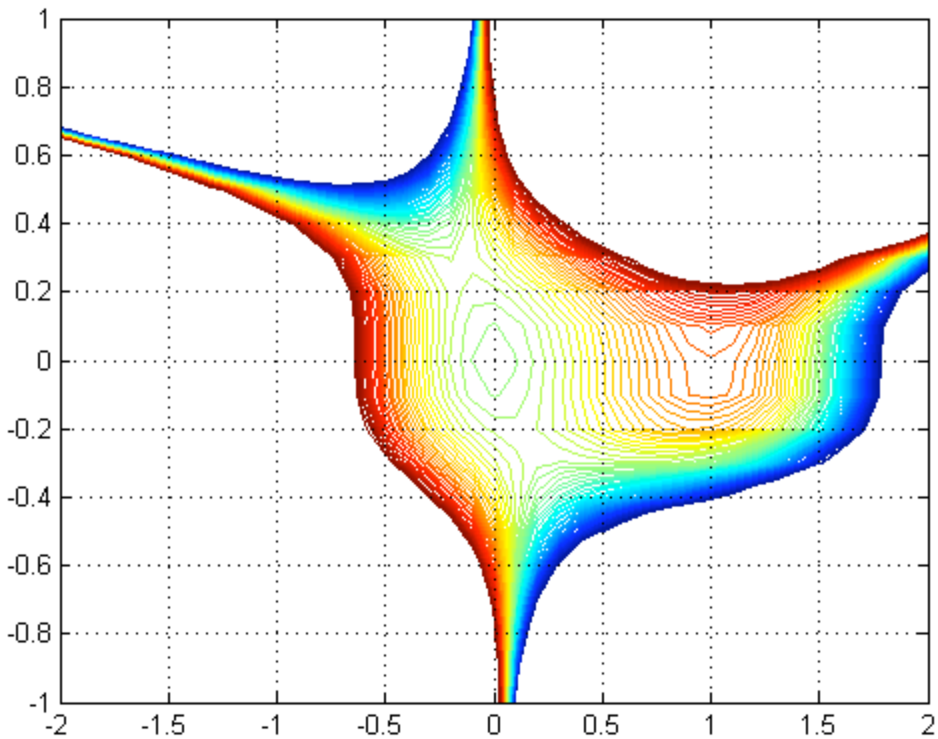
0.1000



```
b=.2

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

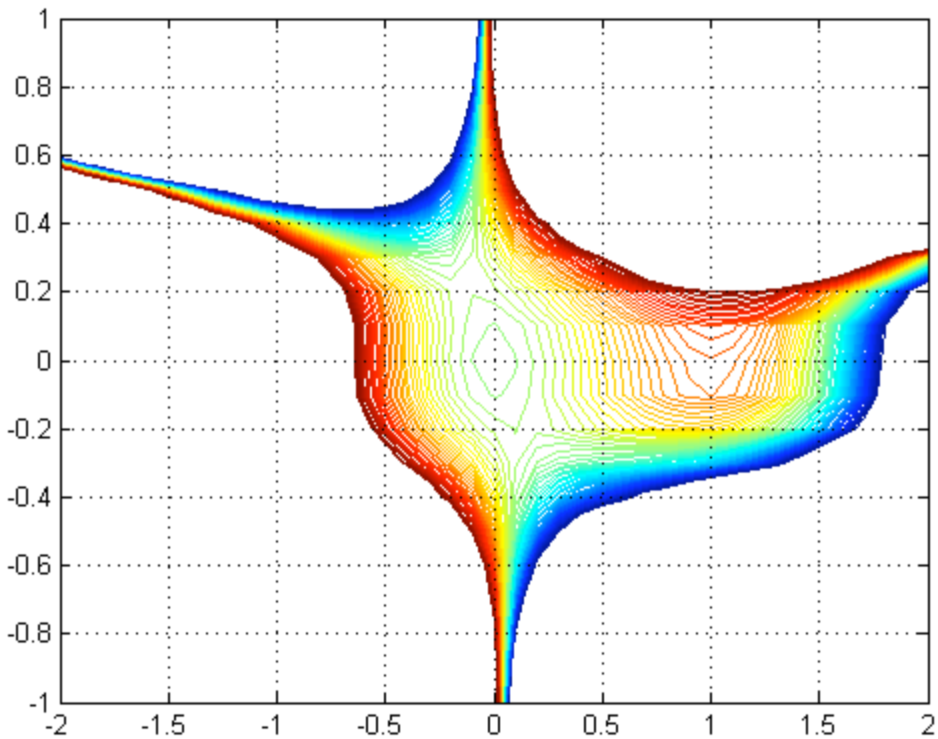
0.2000



```
b=.3

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

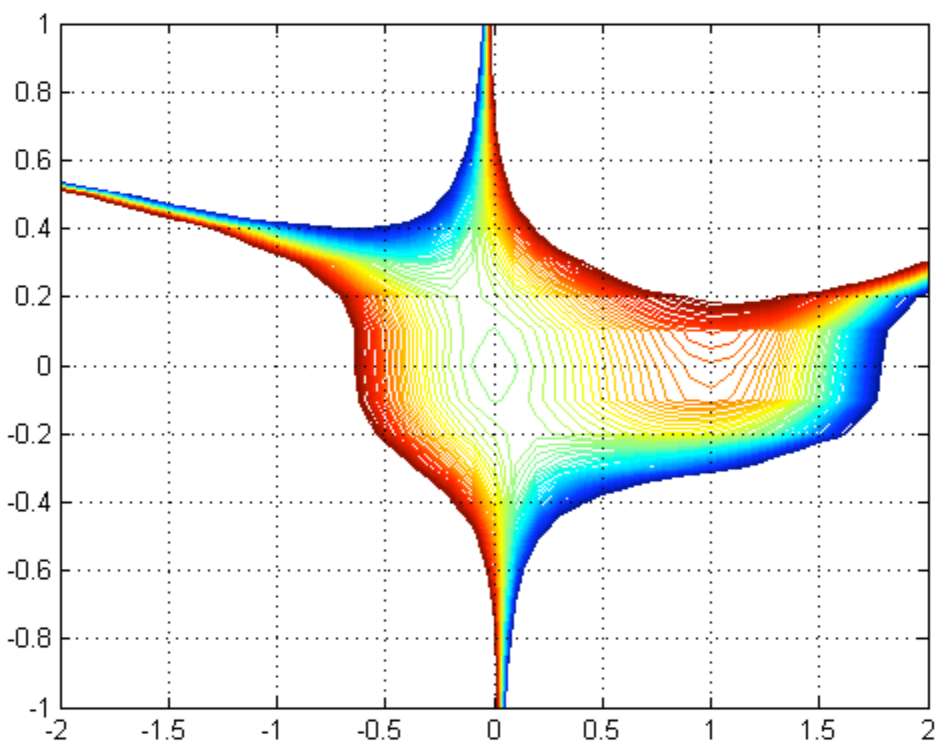
0.3000



```
b=.4

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

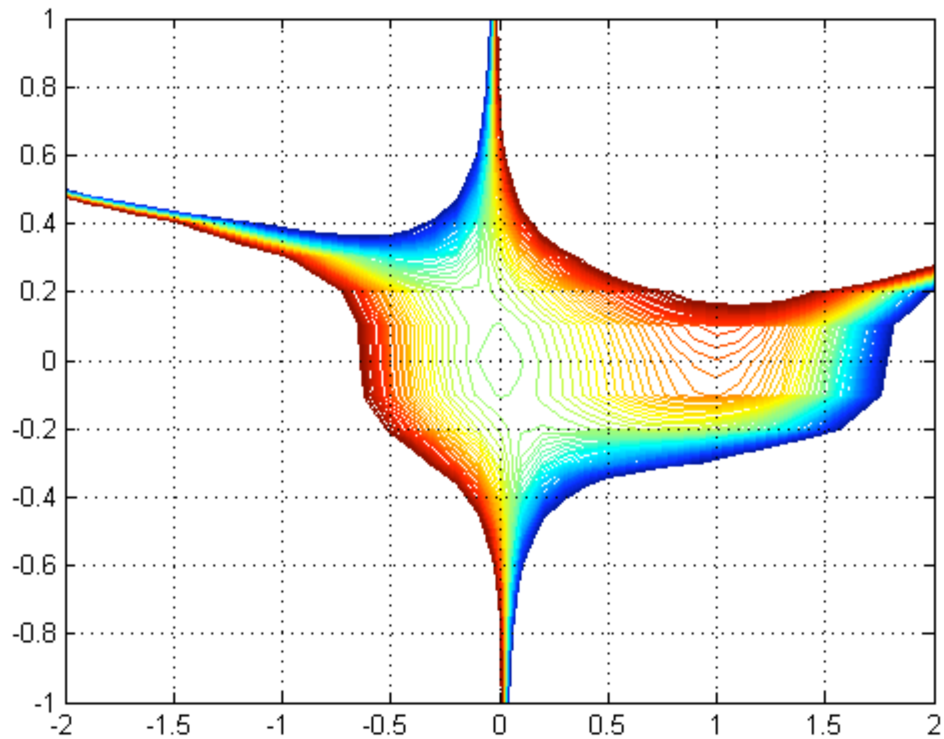
0.4000



```
b=.5
f=@(x,y)(y.^2/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;
Z=f(X,Y);
level=-.3:.009:.3;
contour(X,Y,Z,level)
axis([-2 2 -1 1])
grid on
```

b =

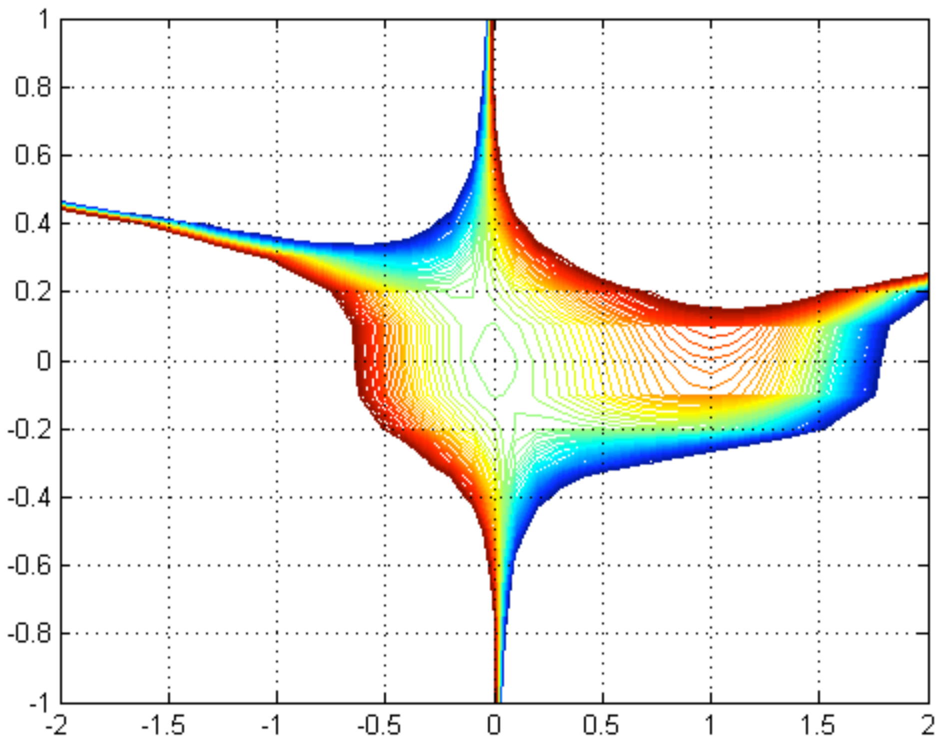
0.5000



```
b=.6

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

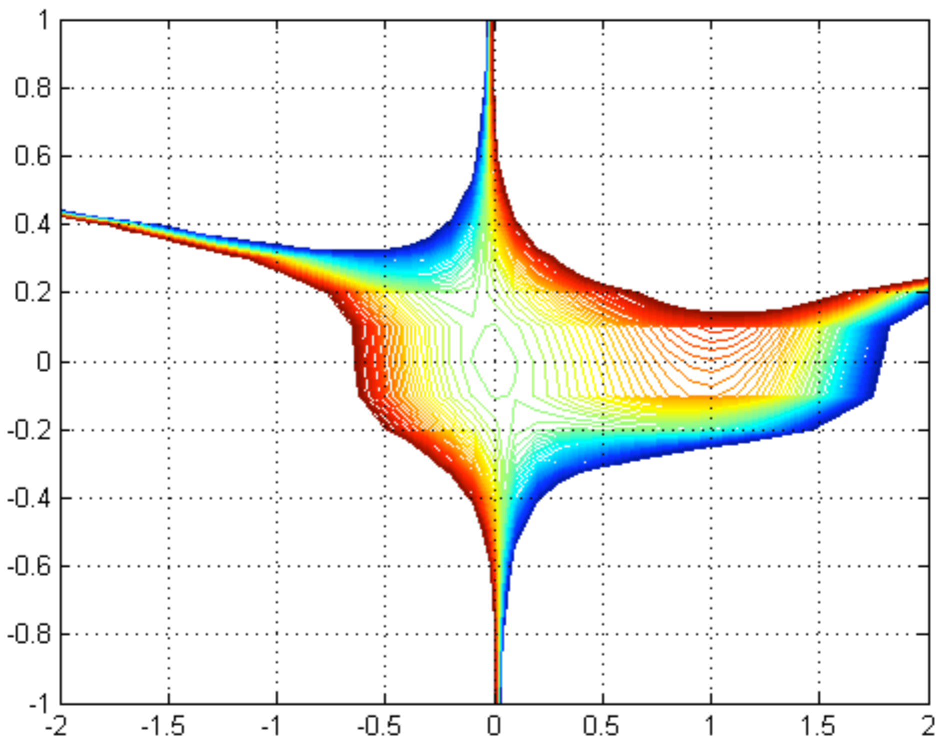
0.6000



```
b=.7

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

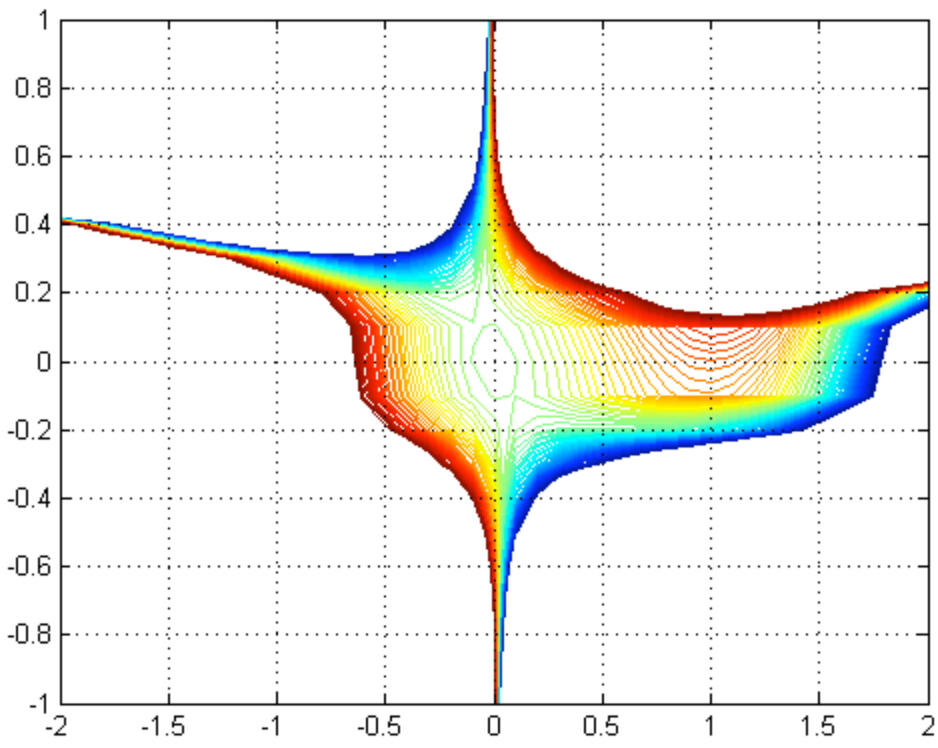
0.7000



```
b=.8

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

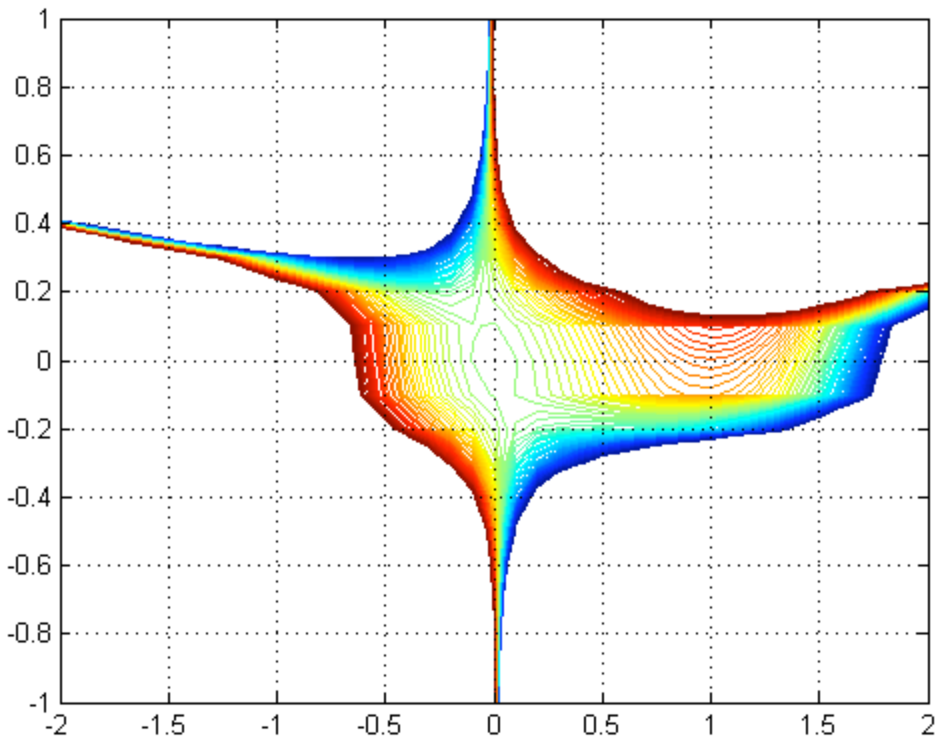
0.8000



```
b=.9

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

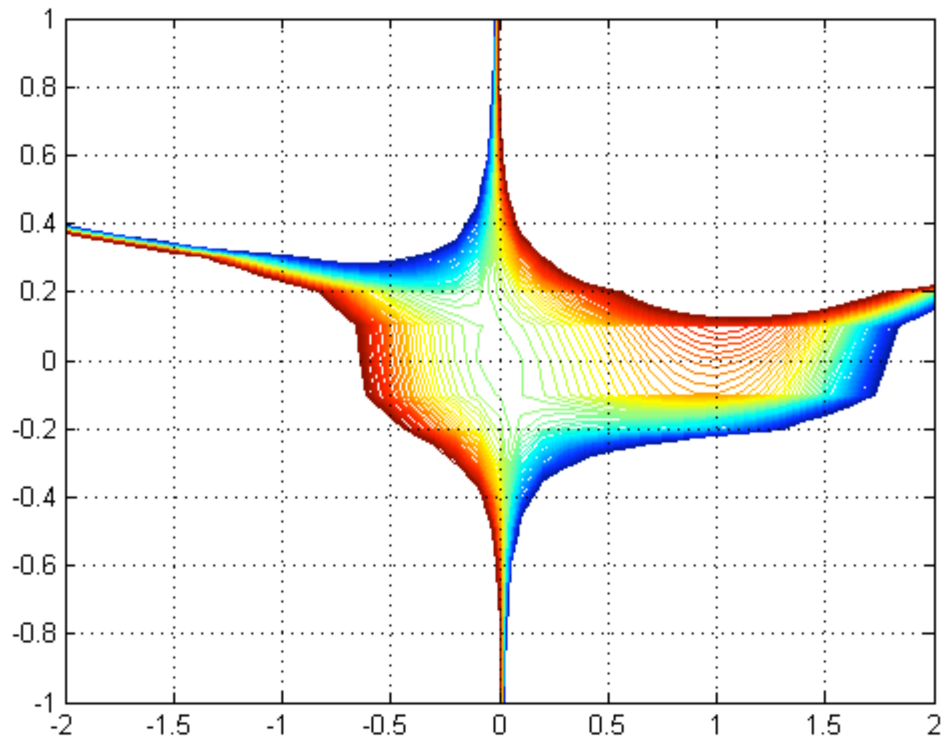
0.9000



```
b=1

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-2 2 -1 1])

grid on
```

b =

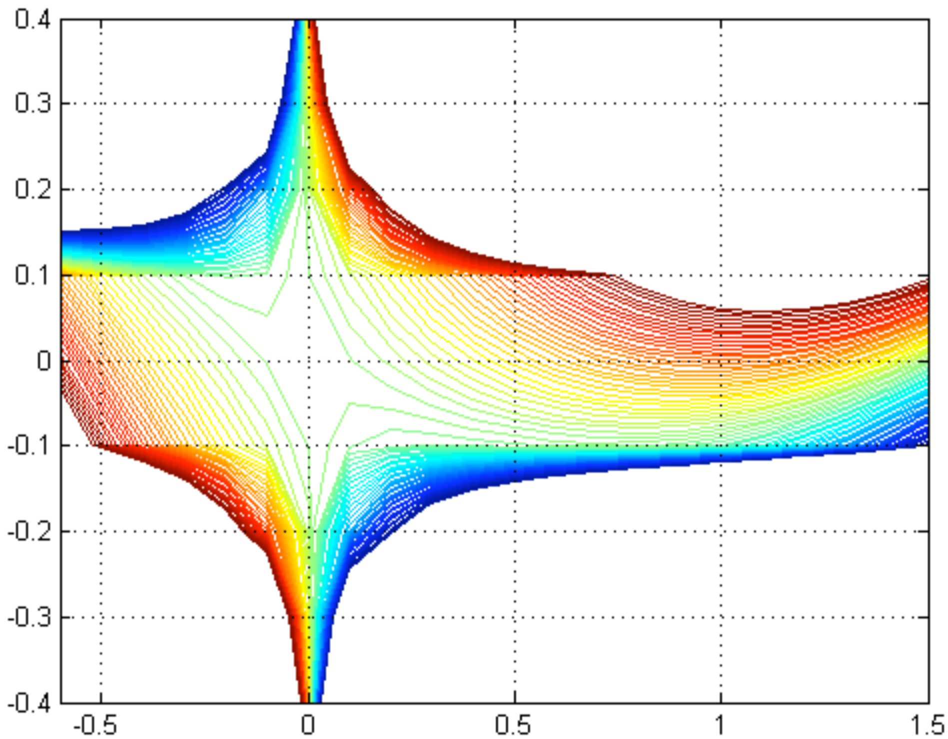
1



```
b=5

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-.6 1.5 -.4 .4])

grid on
```

b =

5



```
b=100

f=@(x,y)((y.^2)/2)+(b*(y.^3)*x)+(x.^2)/2 -(x.^3)/3;

Z=f(X,Y);

level=-.3:.009:.3;

contour(X,Y,Z,level)

axis([-.5 1.5 -.7 .7])

grid on
```

b =

100