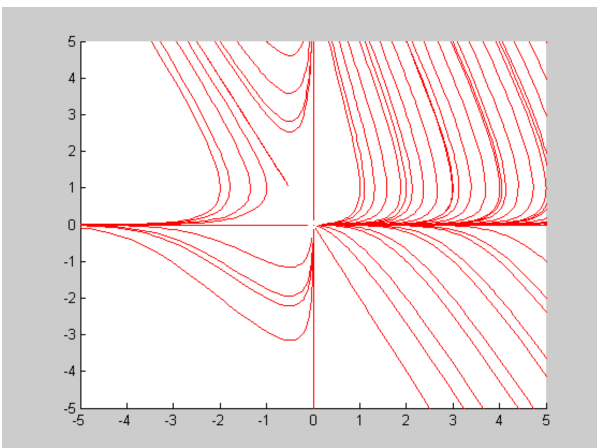
```
_Anh-Duc Van_matlab project_equation 5 and 11 from 9.2

warning off all
for a=0:.1:1
figure; hold on
axis tight
f=@(t,x) [(1-a)*(x(1)-x(1)*x(2))+(a)*(-x(1)+2*x(1)*x(2)); (1-
a)*(x(2)+2*x(1)*x(2))+a*(x(2)-(x(1))^2-(x(2))^2)];
for c=-2:5
     for b=-2:5
         [t,xa]=ode45(f,[0 2],[c,b]);
         plot (xa(:,1),xa(:,2),'r')
         [t,xa]=ode45(f,[0 -2],[c b]);
         plot(xa(:,1),xa(:,2),'r')
     end
      axis([-5 5 -5 5])
 end
end
```

a=0



```
Critical points for a=0
[    0,    0]
[ -1/2,    1]
A =
[   1-y,    -x]
[   2*y, 1+2*x]
evals =
       1
 1+2*x-y
```

Eigenvalues for (0,0)

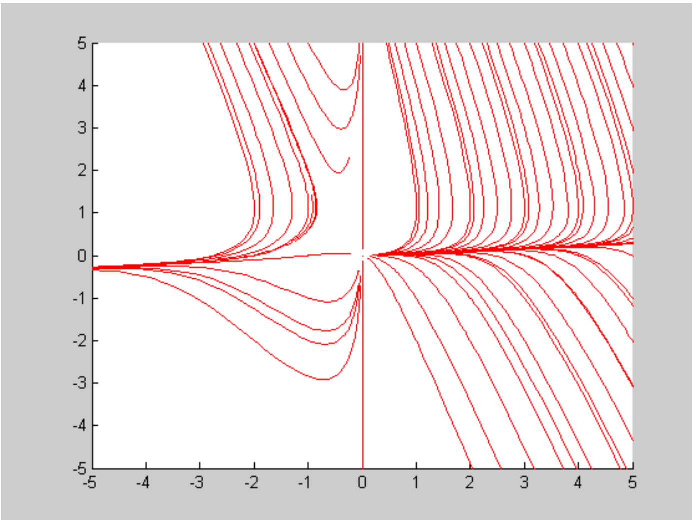
1,1

Eigenvalues for (-1/2,1)

1,-1

When a=0, there are two critical points. At (0, 0) it is a node source and it is unstable. At (-1/2, 1) it is a node sink and it is asymptotically stable.

For a=.1000




```
Critical points for a=0.1000
[                    0,                   0]
[                    0,                  10]
[ 72/7-4/7*355^(1/2),                   8/7]
[ 72/7+4/7*355^(1/2),                   8/7]
A =
[    4/5-7/10*y,        -7/10*x]
[   9/5*y-1/5*x, 1+9/5*x-1/5*y]
evals =
9/10-9/20*y+9/10*x+1/20*(4+20*y+72*x+25*y^2-324*x*y+380*x^2)^(1/2)
9/10-9/20*y+9/10*x-1/20*(4+20*y+72*x+25*y^2-324*x*y+380*x^2)^(1/2)
```

Eigenvalues for (0,0)

1,.800

Eigenvalues for(0,10)

-1,-6.2000

Eigenvalues for ([72/7-4/7*355^(1/2),8/7])

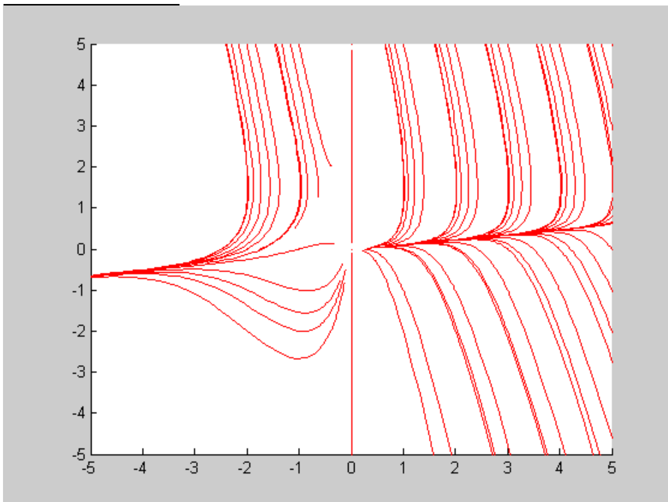
0.8056,-0.8997

Eigenvalues for ([72/7+4/7*355^(1/2),8/7])

39.4695,-0.8040

At a=.1000 there now appears to be 4 critical point. The critical point (0,0) is still there but the critical point (-1/2,1) has disappear and 3 new one has appear. At (0,0) it is still a node source, and it is unstable. At (0,10) it is node sink and it is stable. At ([72/7-4/7*355^(1/2),8/7]) it is a saddle point and it is unstable. At ([72/7+4/7*355^(1/2),8/7]) it is also a saddle point, and it is unstable. At a=.100 the node point is still there but now there appear to be two saddle point.

For a=.2000

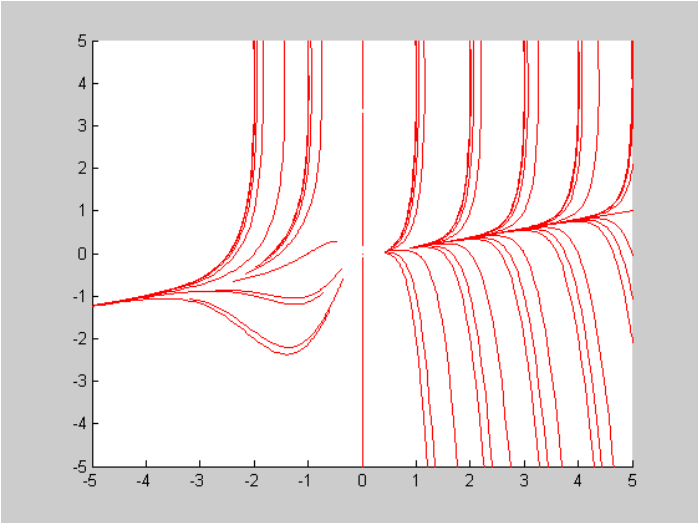



```
Critical points for a=0.2000
[                0,                0]
[                0,                5]
[ 6-1/2*165^(1/2),              3/2]
[ 6+1/2*165^(1/2),              3/2]
A =
[     3/5-2/5*y,          -2/5*x]
[   8/5*y-2/5*x, 1+8/5*x-2/5*y]
evals =
 4/5-2/5*y+4/5*x+1/5*(1+8*x-16*x*y+20*x^2)^(1/2)
 4/5-2/5*y+4/5*x-1/5*(1+8*x-16*x*y+20*x^2)^(1/2)
```

Eigenvalue for (0,0)
1, .600
Eigenvalue for (0,5)
-1,-1.400
Eigenvalue for ([6-1/2*165^(1/2),3/2])
0.5352,-0.8114
Eigenvalue for ([6+1/2*165^(1/2),3/2])
20.8874, -0.6112

At a=.200 there are still 4 critical point, the critical point have all changed excepted (0,0). At (0,0) it is still a node source and it is unstable. At (0,5) it is a node sink and it is sable. The other two are saddle point.

a=.3000

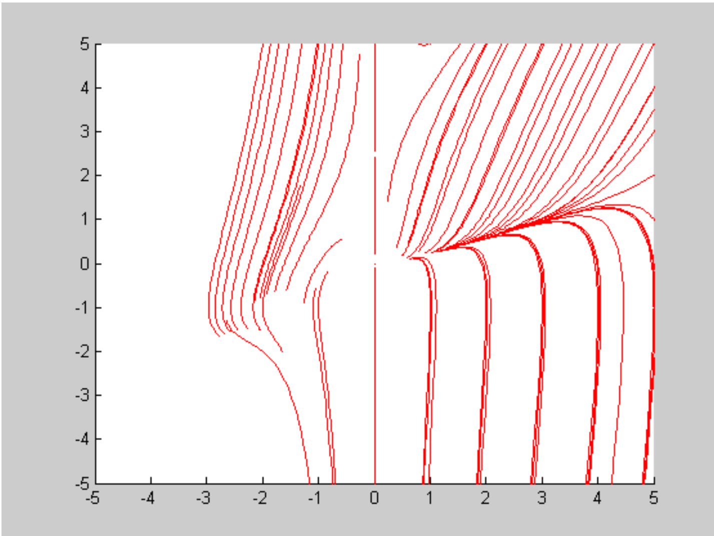



```
Critical points for a=0.3000
[                   0,                    0]
[                   0,                 10/3]
[ 28/3-2/3*190^(1/2),                    4]
[ 28/3+2/3*190^(1/2),                    4]
A =
[    2/5-1/10*y,       -1/10*x]
[   7/5*y-3/5*x, 1+7/5*x-3/5*y]
evals =
7/10-7/20*y+7/10*x+1/20*(36-60*y+168*x+25*y^2-196*x*y+220*x^2)^(1/2)
7/10-7/20*y+7/10*x-1/20*(36-60*y+168*x+25*y^2-196*x*y+220*x^2)^(1/2)
```

<u>Eigenvalue at (0,0)</u>
1, 0.4000
<u>Eigenvalue at (0,10/3)</u>
0.0667,-1.0000
<u>Eigenvalue at ([28/3-2/3*190^(1/2),4])</u>
-0.0704,-1.1281
<u>Eigenvalue at ([28/3+2/3*190^(1/2),4])</u>
24.9412, -0.4095

At a=.300 there are still 4 critical points. At a=0 it is still a node source. At (0,10/3) it is a saddle point. At ([28/3-2/3*190^(1/2),4]) it is a node sink. At [28/3+2/3*190^(1/2),4] it is a node saddle point. It appears that the saddle point is getting closer to the center.

a=.4000

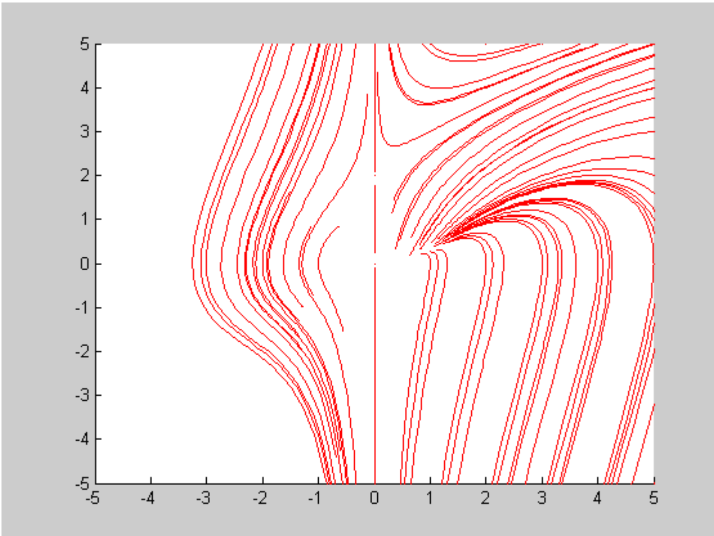



```
Critical points for a=0.4000
[                   0,                   0]
[                   0,                 5/2]
[ -3/2-1/2*i*5^(1/2),                  -1]
[ -3/2+1/2*i*5^(1/2),                  -1]
A =
[      1/5+1/5*y,          1/5*x]
[    6/5*y-4/5*x, 1+6/5*x-4/5*y]
evals =
3/5-3/10*y+3/5*x+1/10*(16-40*y+48*x+25*y^2-36*x*y+20*x^2)^(1/2)
3/5-3/10*y+3/5*x-1/10*(16-40*y+48*x+25*y^2-36*x*y+20*x^2)^(1/2)
```

Eigenvalue for (0,0)
1,.200
Eigenvalue for (0,5/2)
0.7000,-1
Eigenvalue for ([-3/2-1/2*i*5^(1/2),-1])
0.2416 - 1.2261i,-0.2416 - 0.1155i
Eigenvalues for ([-3/2+1/2*i*5^(1/2),-1])
0.2416 + 1.2261i,-0.2416 + 0.1155i

At a=.4 there are 4 critical point. At (0,0) it a node source. At (0,5/2) it is a saddle point. The other two critical point I am not sure what is happening there. There are two imaginary value so I assume it is becoming a spiral but the eigenvalues are not the same so I am not sure what is happening. It could be that at these 2 critical points it is changing to be a center or a spiral.

a=.500

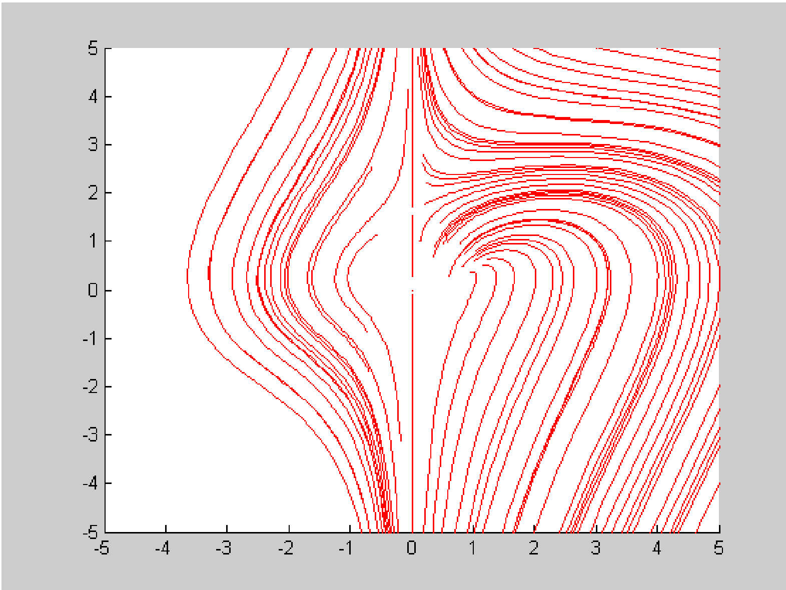


Critical points for a=0.5000
[0, 0]
[0, 0]
[0, 2]
A =
[1/2*y, 1/2*x]
[y-x, 1+x-y]
evals =
1/2-1/4*y+1/2*x+1/4*(4-12*y+8*x+9*y^2-4*x*y-4*x^2)^(1/2)
1/2-1/4*y+1/2*x-1/4*(4-12*y+8*x+9*y^2-4*x*y-4*x^2)^(1/2)

<u>Eigenvalue for (0,0)</u>
1,0
<u>Eigenvalue for (0,0)</u>
1,0
<u>Eigenvalue for (0,2)</u>
1,-1

At a=.500 there are only 3 critical point. It may be that the two weird critical point at a=.400 is merging or somehow making the picture change so that there are only 3 critical points now. This picture is also weird because there are two critical point at the same location. At (0,0) I am not sure what it is but the book said it is a nonisolated critical points. Every point on the line through the eigenvalue is a critical point. At (0,2) it is a saddle point.

a=.600




```
Critical points for a=0.6000
[                  0,                  0]
[                  0,                5/3]
[ 1/6-1/12*55^(1/2),                 1/4]
[ 1/6+1/12*55^(1/2),                 1/4]
A =
[    -1/5+4/5*y,          4/5*x]
[   4/5*y-6/5*x, 1+4/5*x-6/5*y]
evals =
2/5-1/5*y+2/5*x+1/5*(9-30*y+12*x+25*y^2-4*x*y-20*x^2)^(1/2)
2/5-1/5*y+2/5*x-1/5*(9-30*y+12*x+25*y^2-4*x*y-20*x^2)^(1/2)
```

Eigenvalues for (0,0)

1,-.200

Eigenvalues for (0,5/3)

1.1333,-1.0000

Eigenvalues for ([1/6-1/12*55^(1/2),1/4])

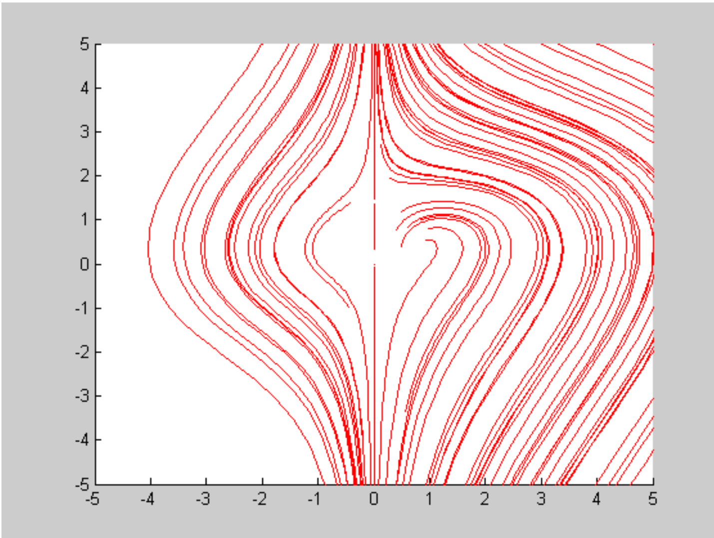
0.1695 + 0.4889i,0.1695 - 0.4889i

Eigenvalues for ([1/6+1/12*55^(1/2),1/4])

0.6639 + 0.1575i,0.6639 - 0.1575i

At a=.600 it is now back to 4 critical point. At (0,0) it is not a saddle point. At (0,5/3) it is saddle point. At ([1/6-1/12*55^(1/2),1/4]) it is a spiral and is unstable. At ([1/6+1/12*55^(1/2),1/4]) it is also a spiral and it is unstable. It seem all the node are now gone and it is just saddle point and spiral.

a=.7000



Critical points for a=0.7000

```
[                    0,                    0]
[                    0,                 10/7]
[ 12/77-2/77*610^(1/2),                 4/11]
[ 12/77+2/77*610^(1/2),                 4/11]
A =
[  -2/5+11/10*y,       11/10*x]
[   3/5*y-7/5*x, 1+3/5*x-7/5*y]
evals =
3/10-3/20*y+3/10*x+1/20*(196-700*y+168*x+625*y^2-36*x*y-580*x^2)^(1/2)
3/10-3/20*y+3/10*x-1/20*(196-700*y+168*x+625*y^2-36*x*y-580*x^2)^(1/2)
```

Eigenvalue for (0,0)

1,-.4000

Eigenvalue for (0,10/7)

1.1714,-1.0000

Eigenvalue for ([12/77-2/77*610^(1/2),4/11])

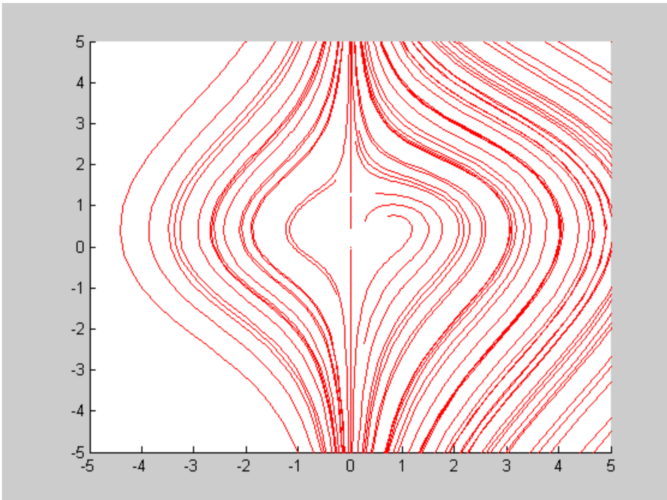
0.0998 + 0.6855i,0.0998 - 0.6855i

Eigenvalue for ([12/77+2/77*610^(1/2),4/11])

0.4847 + 0.7435i,0.4847 - 0.7435i

At a=.700 there are 4 critical point. At (0,0) it is a saddle point. At (0,10/7) it is a saddle point. The other two are spiral source and they are both are unstable. It seems like they are beginning to start forming a circle.

a=.8000




```
Critical points for a=0.8000
[                    0,                    0]
[                    0,                  5/4]
[ 3/28-1/28*285^(1/2),                  3/7]
[ 3/28+1/28*285^(1/2),                  3/7]
A =
[    -3/5+7/5*y,         7/5*x]
[   2/5*y-8/5*x, 1+2/5*x-8/5*y]
evals =
 1/5-1/10*y+1/5*x+1/10*(64-240*y+32*x+225*y^2-4*x*y-220*x^2)^(1/2)
 1/5-1/10*y+1/5*x-1/10*(64-240*y+32*x+225*y^2-4*x*y-220*x^2)^(1/2)
```

Eigenvalue for (0,0)

1,-.6000

Eigenvalue for (0,5/4)

1.1500,-1

Eigenvalue for ([3/28-1/28*285^(1/2),3/7])

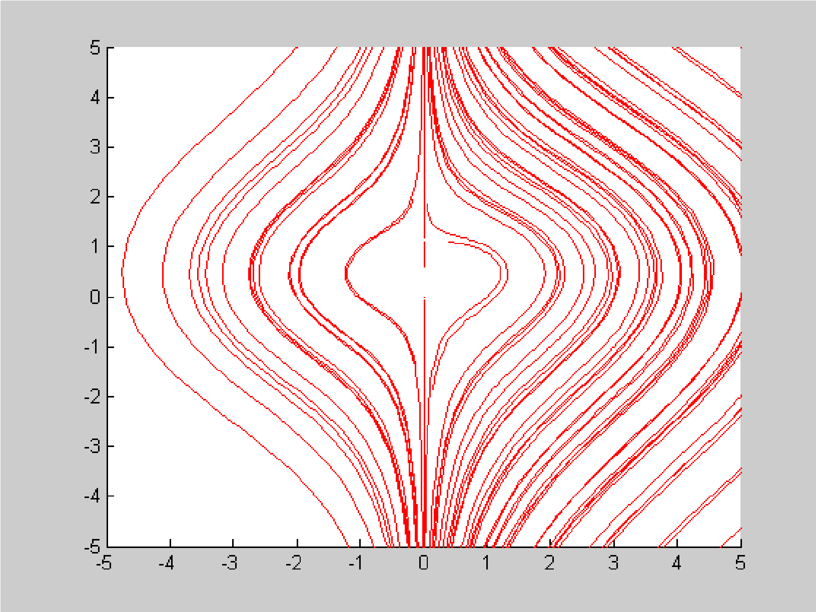
0.0580 + 0.8162i,0.0580 - 0.8162i

Eigenvalue for ([3/28+1/28*285^(1/2),3/7])

0.2992 + 0.9325i,0.2992 - 0.9325i

At a=.8000 there are 4 critical points. At (0,0) and (0,5/4) it is a saddle. The other two is a spiral source, but it seems that the real part of the eigenvalue is getting smaller and the complex part is approaching 1.

a=.9000

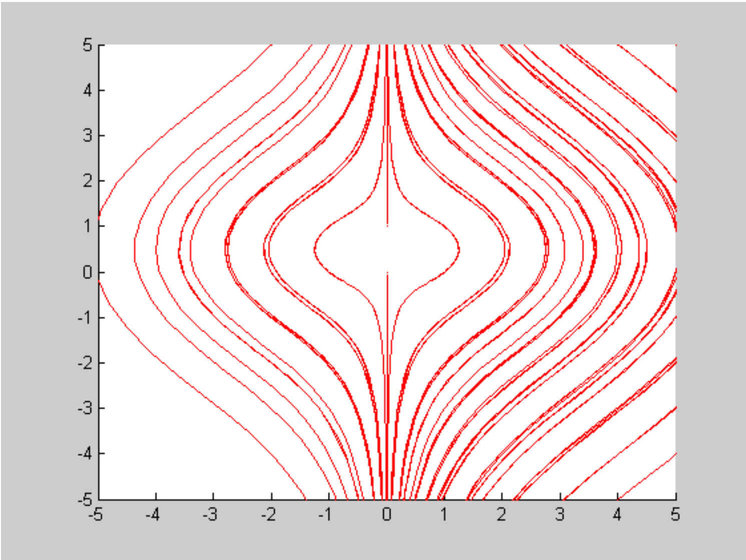



```
Critical points for a=0.9000
[                      0,                       0]
[                      0,                    10/9]
[ 8/153-4/153*445^(1/2),                    8/17]
[ 8/153+4/153*445^(1/2),                    8/17]
A =
[  -4/5+17/10*y,       17/10*x]
[   1/5*y-9/5*x, 1+1/5*x-9/5*y]
evals =
1/10-1/20*y+1/10*x+1/20*(324-1260*y+72*x+1225*y^2-4*x*y-1220*x^2)^(1/2)
1/10-1/20*y+1/10*x-1/20*(324-1260*y+72*x+1225*y^2-4*x*y-1220*x^2)^(1/2)
```

<u>Eigenvalue for (0,0)</u>
1,-.8000
<u>Eigenvalue for (0,10/9)</u>
1.0889,-1.0000
<u>Eigenvalue for ([8/153-4/153*445^(1/2),8/17])</u>
0.0265 + 0.9175i,0.0265 - 0.9175i
<u>Eigenvalue for ([8/153+4/153*445^(1/2),8/17])</u>
0.1368 + 1.0001i,0.1368 - 1.0001i

At a=.9000 there are 4 critical points. At (0,0) and (0, 10/9) it is a saddle, but the value seems to approaching 1 and -1. The other two critical points are spiral source, and are unstable. The real parts are approaching 0 and the complex is approaching 1 or -1.

a=1.000



Critical points for a=1

```
[    0,    0]
[    0,    1]
[ -1/2,  1/2]
[  1/2,  1/2]
A =
[ -1+2*y,    2*x]
[   -2*x,  1-2*y]
evals =
(1-4*y+4*y^2-4*x^2)^(1/2)
-(1-4*y+4*y^2-4*x^2)^(1/2)
```

<u>Eigenvalue for (0,0)</u>
1,-1
<u>Eigenvalue for (0,1)</u>
1,-1
<u>Eigenvalue for (-1/2,1/2)</u>
0.0   + 1.0000i,-0.0000 - 1.0000i
<u>Eigenvalue for (1/2,1/2)</u>
0.0000 + 1.0000i,-0.0000 - 1.0000i

At a=1 there are 4 critical points now. At (0,0) and (0,1) it is a saddle point and the value are 1 and -1 respectively. The other two are now center because the real part are now 0, and the complex has approach -1 or 1. If you set the value of a from 0 to 1, it changes the number of critical point and seems to make it a saddle and center picture. At the beginning it used to be a node and saddle, and then it begins to change until the node is completely gone and a spiral appear. Even then the spiral begins to change until it becomes a center.