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MATH246
Section 0221
Competing Species vs. Predator-Prey Model

Introduction:
Using mathematical models, engineers can predict and analyze the characteristics of an interacting ecosystem and the role each individual organism fills in the population dynamic. Two commonly used models are the Competing Species Model and the Predator-Prey Model. The Competing Species Model documents the interaction between two similar species in their competition for food, resources, and survival, while the Predator-Prey Model exemplifies a simple two-species food chain. In this project, both models will be combined into a single system with the common parameter of alpha. As the value of alpha is altered between the interval of [0,1] we will be able to see a transition from one model to the next, representing the evolution of an organism in this synthetic ecosystem.

Competing Species System of Equations:
Dx/dt= (1-alpha)*((x*(1.5-x-0.5*y))
Dy/dt= (1-alpha)*(y*(2-0.5*y-1.5*x))

Predator-Prey System of Equations:
Dx/dt= (alpha)*(x*(1-0.5+x))
Dy/dt= (alpha)*(-0.25+0.5*x))

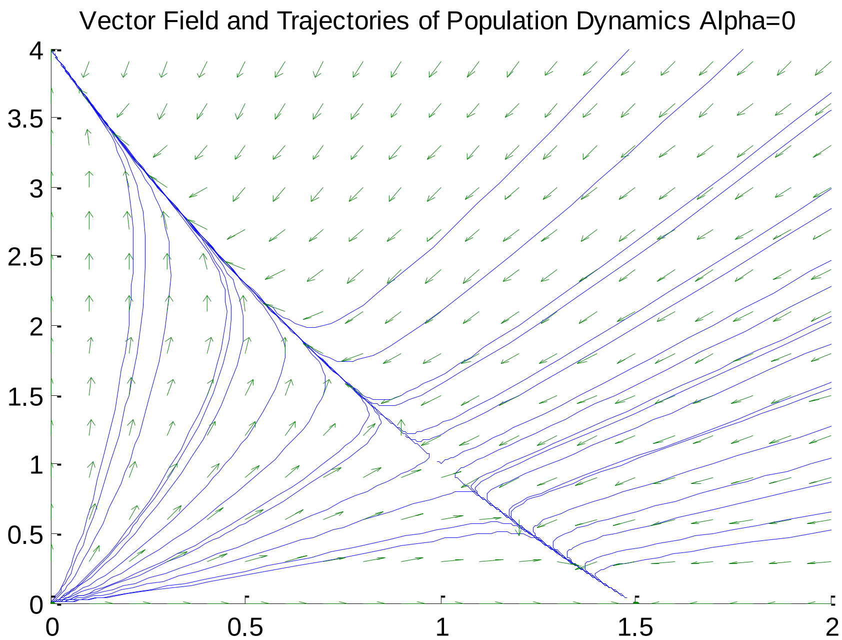
Generic Code
The same MATLAB Code can be used for all the following graphs, with the mere alteration of the value of alpha. The different values of alpha being used are 0, 0.125, 0.25, 0.375, 0.5, 0.625, 0.75, 0.78, 0.875, and 1. The variable "h" will be used in place of alpha. The first portion of the code is used to find the systems critical points. The second portion produces the graph.

```
h=0;
syms x y
sys1=(1-h)*(x*(1.5-x-0.5*y))+(h)*(x*(1-0.5+x));
sys2= (1-h)*(y*(2-0.5*y-1.5*x))+(h)*(y*(-0.25+0.5*x));
[xc,yc]= solve(sys1, sys2, x,y)
disp('Critical Points:'); disp([xc,yc]);
```

```
h=0;
warning off all
f1=@(t,x)[(1-h)*(x(1)*(1.5-x(1)-0.5*x(2)))+((h)*(x(1)*(1-0.5+x(1))))];
f2=@(t,x)[(1-h)*(x(2)*(2-0.5*x(2)-1.5*x(1)))+(h)*(x(2)*(-0.25+0.5*x(1)))];
f3=@(t,x)[(1-h)*(x(1)*(1.5-x(1)-0.5*x(2)))+((h)*(x(1)*(1-0.5+x(1))));((1-
h)*(x(2)*(2-0.5*x(2)-1.5*x(1))))+((h)*(x(2)*(-0.25+0.5*x(1))))];
figure; hold on
for a = 0.20:0.25:2
    for b = 0.5:0.5:2
        [t,xa] = ode45(f3, [0 10], [a b]);
        plot(xa(:,1), xa(:,2))
        [t,xa] = ode45(f3, [0 -5], [a b]);
        plot(xa(:,1), xa(:,2))
```

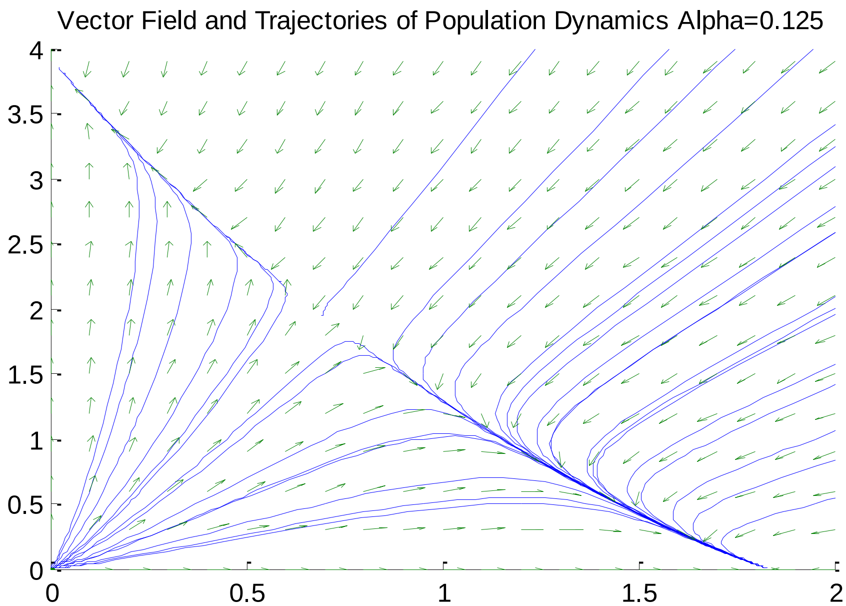
```
    end
end
axis([0 2 0 4])

hold on
[X,Y] = meshgrid(0:0.1:2, 0:0.3:4.5);
U = (1-h)*(X.*(1.5-X-0.5*Y))+(h)*(X.*(1-0.5+X));
V = (1-h)*(Y.*(2-0.5*Y-1.5*X))+(h)*(Y.*(-0.25+0.5*X))
L = sqrt((U/2).^2 + (V/4).^2);
quiver(X,Y, U./L, V./L, 0.4);
axis ([0 2 0 4])
title 'Vector Field and Trajectories of Population Dynamics Alpha=0'
```

Analysis:



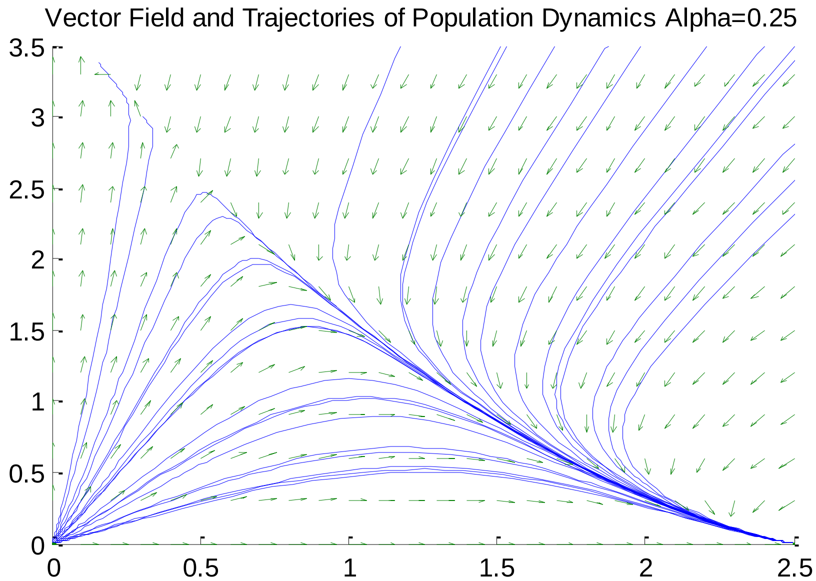

```
Critical Points:
[   0,    0]
[   0,    4]
[ 3/2,    0]
[   1,    1]
```

When the value of alpha is set at 0, the system is unaffected by the predator-prey model and exhibits the characteristics of a competing species model. The first critical points represent a time of extinction for both species, while the second and third critical point represents the extinction of Species X or Species Y. Methods of extinction are limited to natural causes such as food scarcity, inefficient hunting techniques, or climate changes, because the predator-prey relationship is non-existent. The fourth critical point represents the time at which both species thrive.



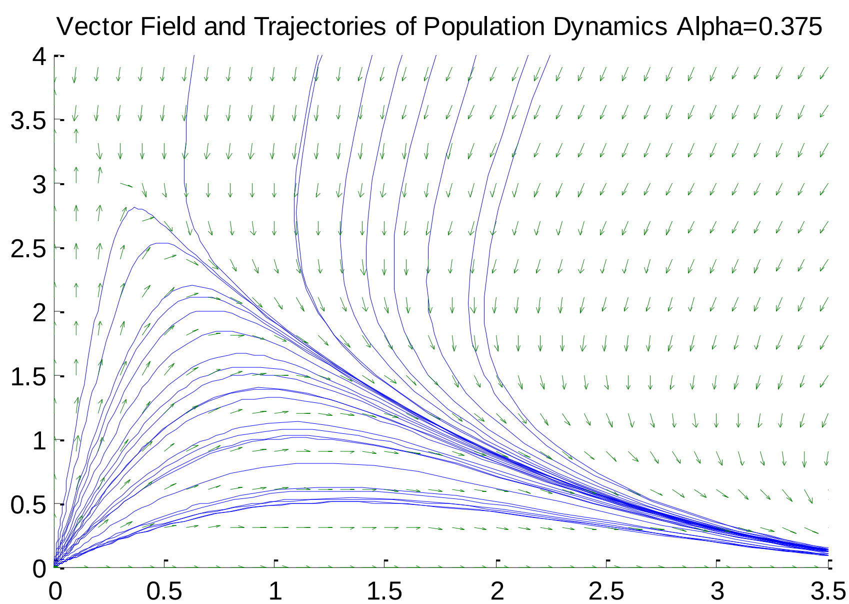

```
Critical Points:
[      0,      0]
[      0,  55/14]
[   11/6,      0]
[  11/16,  55/28]
```

As the alpha value is slightly increased to 0.125, the phase-portrait appears to widen, but the change is very minimal. The time at which both species die remains the same, but the time at which Species X dies off is increased to 1.83, while the time at which Species Y dies off has decreased to the value of 3.93.




```
Critical Points:
[     0,     0]
[     0, 23/6]
[   5/2,     0]
[   3/8, 17/6]
```

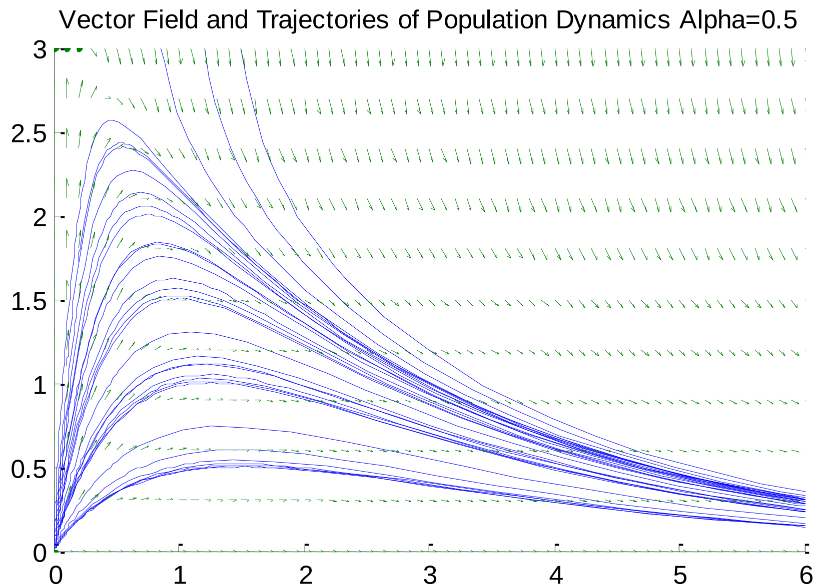
The alpha value is raised again by an interval of 0.125 to the value of 0.25, and there is a little change in the shape of the phase portrait. It begins to narrow at the bottom right, while the extinction time of Species Y continues to lower to 3.83, and the extinction time of Spceies X lengthens to 2.5.




```
Critical Points:
```

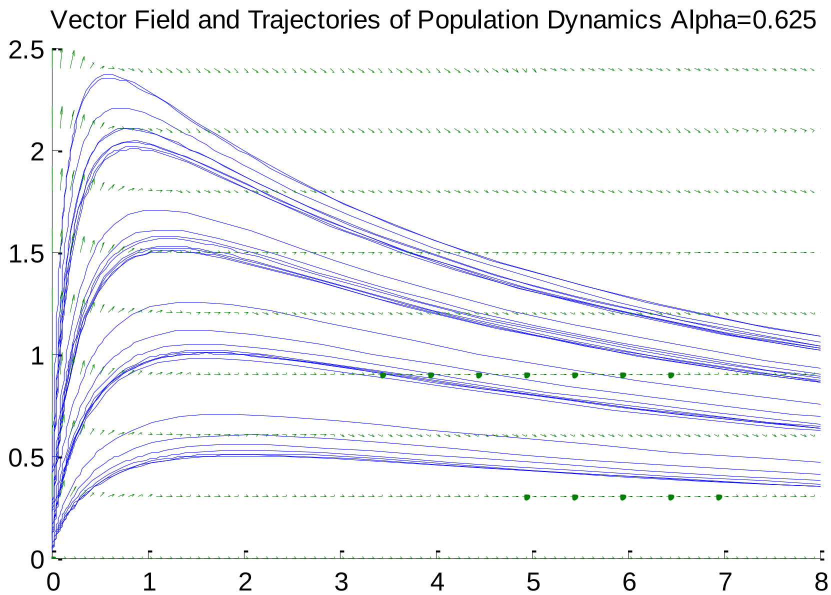
```
[      0,      0]
[      0,  37/10]
[    9/2,      0]
[   1/16,  71/20]
```

This graph shows the phase portrait when alpha= 0.375. The model really begins to illustrate a funneling effect, showing the systems slow transition from a competing species to a predator-prey system. The critical point representing Species Y extinction time continues to decrease, while the critical point of Species X also continues to shift upward.



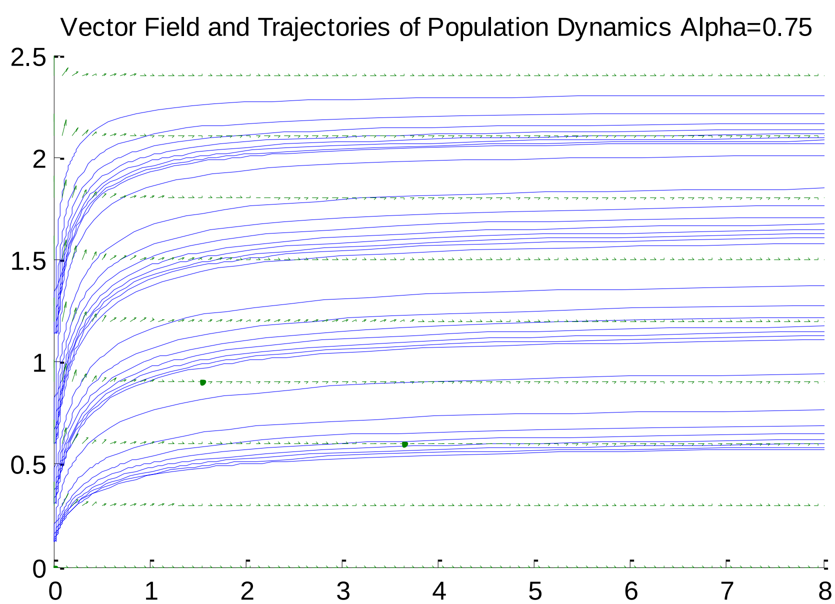

```
Critical Points:
[     0,     0]
[     0,   7/2]
[  -1/4,     4]
```

When alpha=0.5, this is the midpoint of the interval. The competing species model is half-way through its transition to a predator-model, where the critical point that represents the time at which both species thrive has disappeared. This signifies that both species are unable to thrive at the same time, which is characteristic of the predator prey model. As one species thrives, the other begins to die off. Since Species Y has shown the continual trend of decreasing, we can suggest that Species Y is the prey. Species X is the predator, since it exhibits a trend of increasing in survival time.




```
Critical Points:
[      0,      0]
[      0,   19/6]
[   -7/2,      0]
[  -9/16,  47/12]
```

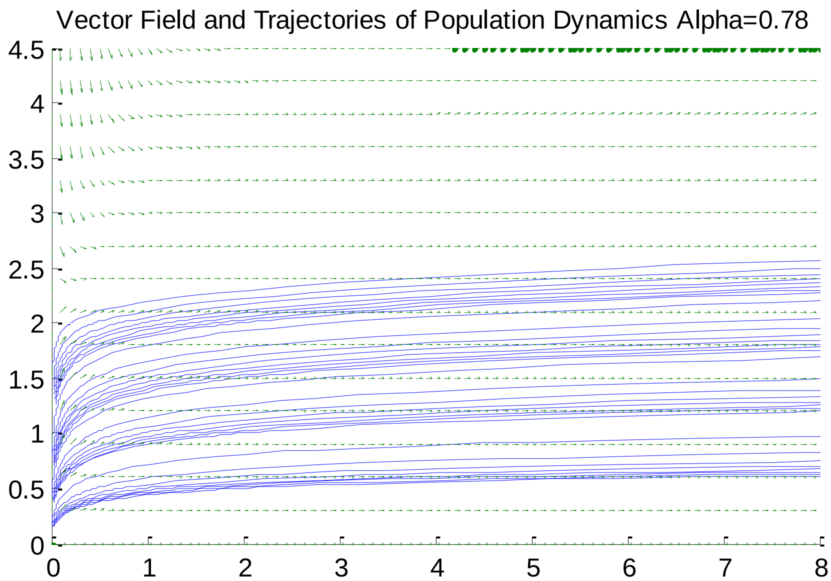
The alpha value has been increased to 0.625, and the phase portrait continues to change, narrowing into 4 defined bands of a predator-prey phase portrait. Species Y continues to die off earlier and earlier, while Species X continues to thrive.



Critical Points:

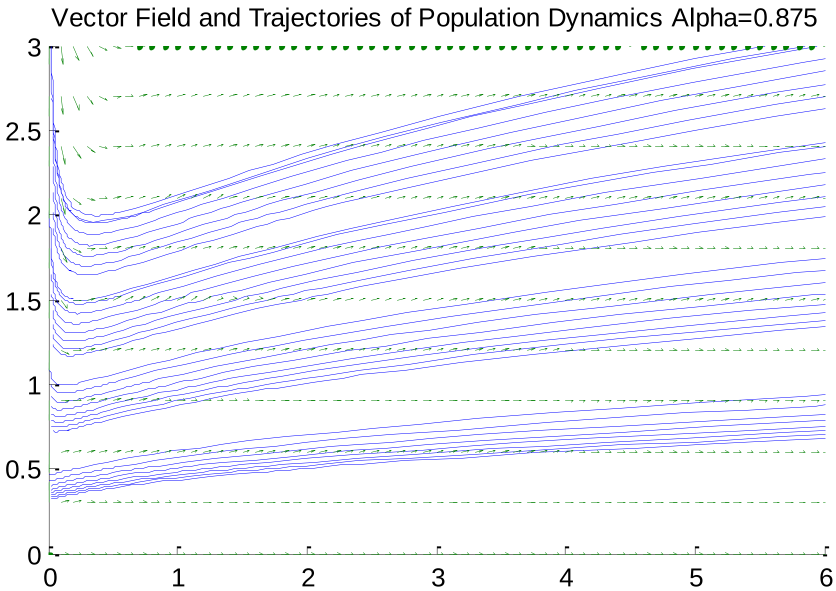
```
[     0,     0]
[ -3/2,     0]
[     0,   5/2]
[ -7/8,   5/2]
```

When alpha is further increased to 0.75, the definition of the peak close to the y-axis has broadened significantly, and the phae-portrait no longer assumes the general shape of a competing species model, but begins to look like a fragment of the predator-prey model. The usual trends in Species X and Y extinction times is constant.



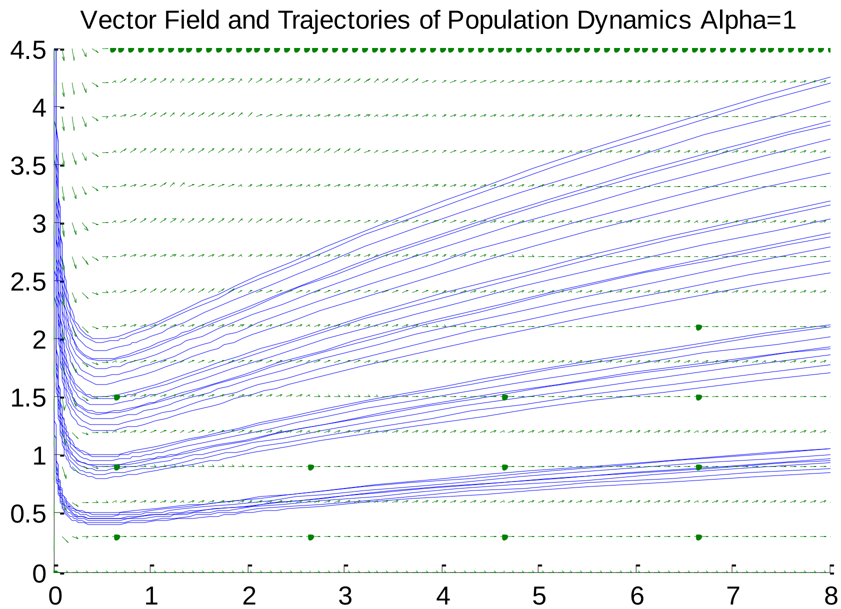

```
Critical Points:
[       0,       0]
[       0,   49/22]
[    -9/7,       0]
[  -19/20,   94/55]
```

The alpha value in this graph varies from the usual 0.125 interval, and instead is only a 0.03 step. This was done in attempts to find an accurate point in which the graph became almost completely linear within the given axis. But instead, the phase portrait remains slightly rounded near the y-axis, showing remnants o f the competing species model.




```
Critical Points:
[      0,       0]
[      0,     1/2]
[   -5/6,       0]
[ -19/16,   -17/4]
```

Here, the alpha value is equal to 0.875, very close to the full transformation to a predator-prey model. The phase portrait no longer is rounded downward, but becomes concave upward near the y-axis, showing just how close the transformation is to being complete. Species Y is at its shortest survival time.




```
Critical Points:
[    0,    0]
[ -1/2,    0]
```

With an alpha=1, the transformation from competing species to predator-prey is complete. The competing species model plays no role in this phase-portrait. If the axis was extended, the phase-portrait would probably show the typical closed-curve shape, with the bottom round shown near the point (0,0). Species Y has become completely extinct, since the critical point no longer exists.

<u>Conclusion</u>:

As shown by the family of phase-portraits, the models of competing species and predator-prey relations ships can be combined into a unifying system. By doing so, we can study the drastic effects each system plays in an ecosystem. If values of alpha were able to surpass the value of 1, it is most likely that the system would repeat itself, since the predator-prey system usually experiences oscillations, which represent the alternating times of success for each species. The combination of both systems actually mimics an ideal ecosystem more accurately; since it is unlikely one relationship would exist without the other.