Homework Problems on the Green Function Method II
Spring 2009, Math 246, Professor David Levermore

1. Compute the Green function $G(t,s)$ for the differential operator $\mathrm{L}(t)$ defined by

$$\mathrm{L}(t)y = \mathrm{D}^2 y - 2t\mathrm{D}y + (t^2-1)y\,,$$

given that $e^{\frac{1}{2}t^2}$ and $te^{\frac{1}{2}t^2}$ solve the homogeneous equation $\mathrm{L}(t)y = 0$. Use the result to solve the initial-value problem

$$y'' - 2ty' + (t^2-1)y = t^2 e^{\frac{1}{2}t^2}\,, \quad y(0) = y'(0) = 0\,.$$

2. Compute the Green function $G(t,s)$ for the differential operator $\mathrm{L}(t)$ defined by

$$\mathrm{L}(t)y = t\,\mathrm{D}^2 y + (t-1)\mathrm{D}y - y\,,$$

given that $t-1$ and e^{-t} solve the homogeneous equation $\mathrm{L}(t)y = 0$. Use the result to solve the initial-value problem

$$t\,y'' + (t-1)y' - y = 2t^3\,, \quad y(1) = y'(1) = 0\,.$$

Remark: You can also solve these problems using variation of parameters.