Quiz 3 Solutions, Math 246, Professor David Levermore
Tuesday, 17 February 2009

(1) [2] Suppose you are using the Runge-Kutta method to numerically approximate the solution of an initial-value problem over the time interval $[0, 5]$. By what factor would you expect the global error to decrease when the number of time steps that you take is increased from 500 to 2500.

Solution: When you increase the number of time steps by a factor of 5, the time step h is reduced by a factor of 5. Because the Runge-Kutta method is fourth order, the global error will therefore decrease by a factor of $5^4 = 625$.

(2) [4] Consider the following MATLAB function M-file.

```
function [t,y] = solveit(ti, yi, tf, n)

h = (tf - ti)/n;
t = zeros(n + 1, 1);
y = zeros(n + 1, 1);
t(1) = ti;
y(1) = yi;
for j = 1:n
t(j + 1) = t(j) + h;
y(j + 1) = y(j) + h*(t(j)^3 + y(j)^2);
end
```

(a) What initial-value problem is being approximated numerically?
(b) What numerical method is being used?

Solution: The initial-value problem being approximated is

$$\frac{dy}{dt} = t^3 + y^2 , \qquad y(t_i) = y_i .$$

The explicit Euler method is being used.

(3) [4] Consider the initial-value problem

$$\frac{dz}{dt} = z^2 - z , \qquad z(0) = 2 .$$

Use the Heun-midpoint method with $h = .1$ to approximate $z(.1)$. Leave your answer as an arithmetic expression.

Solution: Set $z_0 = z(0) = 2$. The Heun-midpoint method then yields

$$f_0 = z_0^2 - z_0 = 2^2 - 2 = 4 - 2 = 2 ,$$

$$z_{\frac{1}{2}} = z_0 + \frac{h}{2} f_0 = 2 + .05 \cdot 2 = 2.1 ,$$

$$f_{\frac{1}{2}} = z_{\frac{1}{2}}^2 - z_{\frac{1}{2}} = (2.1)^2 - 2.1 ,$$

$$z(.1) \approx z_1 = z_0 + h\, f_{\frac{1}{2}} = 2 + .1\big((2.1)^2 - 2.1\big) .$$