

**Homework Problems on the Green Function Method II**  
**Fall 2009, Math 246, Professor David Levermore**

1. Compute the Green function  $G(t, s)$  for the differential operator  $L(t)$  defined by

$$L(t)y = D^2y - 2tDy + (t^2 - 1)y,$$

given that  $e^{\frac{1}{2}t^2}$  and  $te^{\frac{1}{2}t^2}$  solve the homogeneous equation  $L(t)y = 0$ . Use the result to solve the initial-value problem

$$y'' - 2ty' + (t^2 - 1)y = t^2e^{\frac{1}{2}t^2}, \quad y(0) = y'(0) = 0.$$

2. Compute the Green function  $G(t, s)$  for the differential operator  $L(t)$  defined by

$$L(t)y = tD^2y + (t - 1)Dy - y,$$

given that  $t - 1$  and  $e^{-t}$  solve the homogeneous equation  $L(t)y = 0$ . Use the result to solve the initial-value problem

$$ty'' + (t - 1)y' - y = 2t^3, \quad y(1) = y'(1) = 0.$$

**Remark:** You can also solve these problems using variation of parameters.