Quiz 4 Solutions, Math 220, Professor David Levermore
Friday, 24 September 2010

(1) [4] Let $f(p)$ be the number of cars sold when the price is p dolars per car. Interpret the statements $f(10,000) = 200,000$, and $f'(10,000) = -3$.

Solution. The statment $f(10,000) = 200,000$ means that

you will sell 200,000 cars if they are priced at 10,000 dollars each.

The statment $f'(10,000) = -3$ means that

you will sell 3 less cars for each dollar that you raise the price of one.

(2) [3] Sketch the graph of a function $f(x)$ over the interval $-1 \leq x \leq 1$ such that $f(x)$ is positive, decreasing, and concave up over that interval.

Solution. Your graph should lie above the x-axis over the interval $-1 \leq x \leq 1$, it should decrease as you move from left to right, and it should bend upwards.

Remark. Examples of formulas that would lead to such a graph are

$$ f(x) = \frac{1}{x+2}, \qquad f(x) = (x-2)^2. $$

The first is a shift of $1/x$ to the left by 2, while the second is a shift of x^2 to the right by 2.

(3) [3] Consider the function $f(x) = 3x^4 - 4x^3 - 6x^2 + 12x$. Its derivative is given by $f'(x) = 12x^3 - 12x^2 - 12x + 12 = 12(x+1)(x-1)^2$. (You do not have to verify this.) Where f increasing? Where is it decreasing?

Solution. The critical points of $f(x)$ are found by setting $f'(x) = 0$. Because $f'(x) = 12x^3 - 12x^2 - 12x + 12 = 12(x+1)(x-1)^2$, the critical points are $x = -1$ and $x = 1$. A sign analysis of $f'(x)$ yields

$$ \begin{array}{ccccc} - & & + & & + \\ \hline & -+ & & -+ & \end{array} $$

decreasing -1 increasing 1 increasing

Therefore f is increasing over $-1 < x < \infty$, and decreasing over $-\infty < x < -1$.

Remark. Notice that $x = 1$ is an absolute minimum.